Chapter Review Sheets for
Elementary Differential Equations and Boundary Value Problems, 10e

Chapter 9: Nonlinear Differential Equations and Stability

Definitions:
• Equilibrium Solutions
• Critical Points
• Trajectory
• Phase Plane
• Phase Portrait
• Node; Nodal Sink, Nodal Source, Saddle Point, Proper Node, (Star Point) Improper Node, (Degenerate Node), Spiral Sink, Spiral Source
• Autonomous Stable, Unstable Isolated Critical Point
• Locally Linear System
• Basis of Attraction
• Globally Asymptotically Stable
• Region of Asymptotic Stability, Nullclines
• Separatrix
• Liapunov's Second Method
• Positive Definite, Negative Definite, Positive Semidefinite, Negative Semidefinite
• Limit Cycle
• Asymptotically Stable
• Strange Attractors
• Chaotic System/Equation

Theorems:
• Theorem 9.3.1: Stability of critical points of linear systems dependence on eigenvalues. (p. 519)
• Theorem 9.3.2: Stability of critical points of almost linear systems. (p. 523)
• Theorem 9.6.1: Stability of critical points dependence on negative definite and negative semidefinite nature of the Liapunov function, and its derivative. (p. 558)
• Theorem 9.6.2: Conditions on definiteness for an unstable critical point. (p. 558)
• Theorem 9.6.3: Conditions for positive definite. (p. 560)
• Theorem 9.6.4: Conditions for \( V(x, y) = ax^2 + bxy + cy^2 \) to be positive or negative definite. (p. 561)
• Theorem 9.7.1: Existence of closed trajectories (p. 568)
• Theorem 9.7.2: Nonexistence of closed trajectories. (p. 568)
• Theorem 9.7.3: Poincaré-Bendixson Theorem. (p. 569)

Important Skills:
• Be able to determine the phase plane and phase portraits of a 2 by 2 linear system.
  The solutions will depend on eigenvalues. Pages 497 - 504 cover the five important cases.
  Table 9.1.1 on page 504 summarizes the eigenvalue results.
• Determine the trajectories for a system of ODE's. (Ex. 3 & 4, p. 514 - 515)
• Know how to determine whether a system of ODE's is locally linear. (Ex. 1 & 2, p. 521)
• Be able to determine the linear system associated with the almost linear system. (Ex. 3, p. 523)
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- Relating the ODE system to the possible motions of a pendulum (Ex. 4, p. 526)
- Sketch phase portraits for competing species. (Ex. 1 or 2, p.532 and p.536)
- Sketch phase portraits for predator-prey. (Ex. 1, p.546)
- Use Liapunov's method to determine the stability of a critical point. (Ex. 1, p.557; Ex. 2, p.559)
- Determine periodic solutions of systems of ODE's. (Ex. 1, p.566)
- Study the solution of the van der Pol equation. (Ex. 2, p. 570)

Relevant Applications:
- Population Modeling, Competing Species, Predator-Prey Modeling