The Heart of Mathematics: An invitation to effective thinking

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KIT BOOKLET

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Please note that this booklet also contains instructions for using the applets on the Interactive Explorations CD, packaged with every new text. Each time the CD icon appears in the text, you will find an applet illustrating the concept being presented.
Mathematics is everywhere in your everyday world. This kit and booklet provide you with a broad sampling of experiments and materials. The intent of these materials is to demonstrate that mathematics and mathematical thinking are all around you. Using materials from your desk, dorm room, or toolbox, you can create models to help you visualize and think about important or practical things in your life. It is our hope that the materials presented here stimulate you to create other models from everyday materials so you can think about issues that confront you. We’d like it if you would e-mail your original activity and puzzle ideas to us for possible use in future editions of our book and kit. E-mail your ideas to: heartofmath@keycollege.com

SCATTERPLOT—Graphically view data and search for patterns, as described in Section 7.6, “What the Average American Has.”

BAR CHART—Collect data about various categories, and then use this activity to view your data graphically. This is yet another way to visualize data, as described in Section 7.6, “What the Average American Has.”

HISTOGRAM AND BOX PLOT—This activity displays data about Old Faithful, the geyser in Yellowstone National Park. Compare the histogram with the one on page 588 in Section 7.6, “What the Average American Has.”

PIE CHART—Try plotting the donut data from page 589 in Section 7.6, “What the Average American Has,” on this bar chart. Collect data about various categories, and then use this activity to view it graphically.

BOX MODEL—You can read about the Law of Large Numbers on page 526 in Section 7.2, “Predicting the Future in an Uncertain World.” In this activity, you can witness this law for yourself. See how successive repetitions lead to mathematical probabilities.

VOTING STRATEGIES: THE WHAMMY AWARDS—Now that you have read about voting strategies in Section 8.4, “Peril at the Polls” (page 683) and learned that the preferred candidate will not always win, use this activity to stage your own elections.

SAVINGS CALCULATOR—You read about becoming a millionaire on page 664 of Section 8.3, “Money Matters.” Now figure out exactly how long it will take you to reach that big goal.

LOAN CALCULATOR—Are you thinking of borrowing money to buy a house or to pay for college? You could visit the One-Day-a-Year Savings and Loan Company, as you read about on page 670 of Section 8.3, “Money Matters,” or you could use this activity.
COBWEB PLOTS—See the cobweb plots as they should be seen: on a computer. Read about them in Mindscape III.30 in Section 6.2, “The Dynamics of Change” (pages 424–425), and again in Mindscapes II.6–18 in Section 6.5 “Predetermined Chaos” (pages 494–498). And you thought your life was chaos!

KOCH SNOWFLAKE AND SIERPINSKI CARPET—Build some cool fractals, as described in Section 6.3, “The Infinitely Detailed Beauty of Fractals” (page 430). They’re incredibly detailed, and it’s fun to watch them evolve.

BARNSLEY FERN AND FRACTAL GARDEN—Watch fractal ferns grow right before your eyes. No need to add water; just add the mathematics from page 440 in Section 6.3, “The Infinitely Detailed Beauty of Fractals.”

POLYGON PROBABILITY FRACTALS—The Chaos Game on page 447 in Section 6.3, “The Infinitely Detailed Beauty of Fractals,” shows how to use randomness to build beautiful fractals. See these ideas unfold in front of your eyes. Pretty amazing stuff.

JULIA AND MANDELBROT SETS—The icons of all fractals are the Julia and Mandelbrot Sets, as shown and described in Section 6.4, “The Mysterious Art of Imaginary Fractals” (page 459). These sets were not discovered until someone viewed them on a computer. Enjoy the infinite detail. Zoom into small parts and discover amazing structure. Build your own Julia Sets, print them out, and hang them on your wall. Really cool.

DUELING CALCULATORS—You may have read about Dueling Calculators on pages 485–487 in Section 6.5, “Predetermined Chaos.” Here you can watch them fight it out. Are calculators always correct? No! See how far off they can actually be.

CONWAY’S GAME OF LIFE—Sure, you can read about the Game of Life on page 414 in Section 6.2, “The Dynamics of Change,” but that’s nothing compared with watching life happen for real. Create your own starting configuration and see if it grows, moves, dies out, or creates chaos. Also, try some of the cool preprogrammed starting configurations. This activity is a must-see.

HAMLET HAPPENS—You read about monkeys typing Shakespeare’s play, Hamlet, on pages 544–546 in Section 7.3, “Random Thoughts.” Now see how long it would really take for the string of letters “To be or not to be” to appear.

COIN TOSSING—You don’t want to flip a penny 100 times; instead, have the computer do it for you. This activity is similar to the penny experiments in Section 7.1, “Chance Surprises.”

Descriptions of Kit Elements

THE ENLARGING AREA PARADOX (also known as The Fibonacci Square-to-Rectangle Puzzle)—This puzzle paradox comprises the following four pieces, found in your kit.

Using the four pieces, first build the square, as shown. Notice that the area of the square is $8 \times 8 = 64$. Next, using the same pieces, construct the rectangle shown. Notice that the area is now $5 \times 13 = 65$. The rectangle seems to have more area, even though no new pieces have been added. Measure the sides of your square and then the sides of your rectangle to actually compute their areas. How can you explain this paradox? The dimensions 5, 8, and 13 happen to be consecutive Fibonacci numbers. See Section 2.2, “Numerical Patterns in Nature,” Mindscape II.15 (pages 59–60). Enjoy.
THE GREAT CIRCLE—Planning a trip from Chicago to Cape Town? Or Honolulu to Moscow? How many miles is that trip? What is the shortest path? You read about this in Section 4.6, “The Shape of Reality?” (page 291); now you can plan your fantasy vacation.

PYTHAGOREAN PUZZLE—Manipulate triangles and rectangles to see a proof of the Pythagorean Theorem. This activity is an interactive version of the material in Section 4.1, “Pythagoras and His Hypotenuse” (page 211).

ART GALLERY—Create your own museums and find out how few cameras you need to see each point. Section 4.2, “A View of an Art Gallery” (page 219), shows you how to solve this problem. Now work it out on the computer.

PLATONIC SOLIDS AND DUALS—The beautifully symmetric Platonic Solids are quite alluring, as revealed in Section 4.5, “The Platonic Solids Turn Amorous” (page 269). Use the CD-ROM to move the solids around to view them from any vantage point. Discover all the views! For an even more exciting view, watch them rotate with their duals.

SLICING SOLIDS WITH A PLANE—Some of the surprising structures of certain solids can be seen by slicing them (see page 321 in Section 4.7, “The Fourth Dimension”). See what happens as you slice up and down. The shapes you get may surprise you.

PATTERN BLOCKS—Symmetric tiling can be very beautiful. Create your own tilings that display the properties discussed in Section 4.4, “Soothing Symmetry and Spinning Pinwheels” (page 250).

TILING THE PLANE WITH THE PINWHEEL PATTERN—The Pinwheel Pattern is unique and fun to explore. In this activity, you can tile the plane with right triangles or build 5-unit, 25-unit, and 125-unit Pinwheel Triangles, just like on page 254 in Section 4.4, “Soothing Symmetry and Spinning Pinwheels.”

SOLIDS AND THE EULER CHARACTERISTIC—So maybe you’ve see the Platonic Solids and verified the Euler Characteristic, as in Section 5.3, “Feeling Edgy?” (page 362). Now experience them live in cyberspace.

FEELING EDGY?—In Section 5.3, “Feeling Edgy?” (pages 360–361), you learned how to predict V − E + F for any squiggle drawn on a piece of paper. Now create more masterpieces and check the Euler Characteristic for your own works of art.

PYTHAGOREAN PUZZLE PROOF—This puzzle consists of five pieces: four identically sized right triangles and one small square.

Your first task is to use all five pieces to make a square with each side equal to \( c \), the hypotenuse. Next, take on the challenge of using the same five pieces to construct two squares, one with sides \( a \) and the other with sides \( b \). By tackling these challenges, you will have discovered your own proof of the Pythagorean Theorem. See Section 4.1, “Pythagoras and His Hypotenuse,” page 208, for some hints and remarks. Invite your friends to take this Pythagorean challenge.

PLATONIC SOLIDS SET

You can construct the skeleton forms of the Platonic (or regular) solids using the straws and pipe cleaners found in your kit (together with some glue, if you wish). The straws will be the edges, and the pipe cleaners will be bent in half and used to connect the edges at the vertices. The crucial step is to make sure that the edges for each individual solid are all of equal length. If the edges of one of these solids have differing lengths, then your final product will look distorted. For the tetrahedron, cube, and octahedron, we suggest using edge lengths that are half of the straws—that is, cut some straws in half to make the edges for those solids. For the dodecahedron and icosahedron, we suggest you cut some straws in thirds to create suitable edges. Use the cutting template on the next page to help you cut the straws into appropriate, accurate lengths. Simply place the straws under the template and make precise cuts.

Look at the pictures on pages 274–275 (or check out the CD-ROM packaged with your text) to figure out how many edges you need for each solid and, thus, how many straws need to be cut for each solid. Once you’ve cut the appropriate number of straws, cut the pipe cleaners into roughly 3-inch segments (for the three solids with
On the CD-ROM, Packaged with your Text:

COUNTERFEIT COIN—Weigh your own coins as in Story 1 of Section 1.1, “Silly Stories Each with a Moral” (pages 5–6). How many weighings do you really need?

LET’S MAKE A DEAL—In Story 7 of Section 1.1, “Silly Stories Each with a Moral” (pages 10–11), should you stick or switch? Have your computer simulate the game to see what happens. Remember the answer the next time you’re on a game show.

FOUNTAIN OF KNOWLEDGE—Just like Trey Sheik in Story 3 of Section 1.1, “Silly Stories Each with a Moral” (page 7), you can attempt to make 8 ounces of juice with cups of sizes 6 ounces and 10 ounces. Try making other combinations.

DODGE BALL—Simulate this two-player game by playing against the computer, just like Story 5 in Section 1.1, “Silly Stories Each with a Moral” (pages 8–9). You can be Player 1 or Player 2. Find a way to beat the computer.

GOLDEN RATIOS AND RECTANGLES—Experience the Golden Ratio and the Golden Rectangle for yourself. You read about it in Section 2.2, “Numerical Patterns in Nature” (pages 54–55); now you can live it!

FIBONACCI SEQUENCE AND THE GOLDEN RATIO—Use various starting numbers to derive Fibonacci-like sequences, as described in Section 2.2, “Numerical Patterns in Nature” (page 51). What number do all of these sequences approach?

CYBER SUE: CARSON KID’S LITTLE SISTER—Sure, you read about the Carson City Kid on pages 97–98 in Section 2.5, “Public Secret Codes and How to Become a Spy.” Now see the method in action with Cyber Sue. Are you ready to break the code?

RSA CODING SCHEME—You’ve learned about Rivest, Shamir, and Adleman on page 98 in Section 2.5, “Public Secret Codes and How to Become a Spy.” Put their coding scheme to the test with any three-letter word.

ROLLING UP THE REAL LINE—You saw how to roll up the number line on page 195 in Section 3.5, “Straightening Up the Circle.” See the stereographic projection on your computer screen, and watch those infinitely many numbers get rolled up.

COUNTING ALL PAIRS—In Section 3.2, “Comparing the Infinite,” you read about the one-to-one correspondence of the rational numbers and the natural numbers (page 154). Now create your own one-to-one correspondences with the grid provided.

Template for Cutting Straws
longer edges) and 2-inch segments (for the two solids with shorter edges). Then bend each straw in half (you may want to make the pipe cleaners a bit wavy to ensure a snugger fit).

Place one end of a pipe cleaner in one straw and the other end in another straw to form part of a vertex. Again, use the pictures on pages 274–275 as a guide to determine how many edges come together at each vertex and the overall structure of each solid. You might want to place some glue on the pipe cleaners before inserting them in the straws to keep the solids together once they are complete.

A word of caution: Be careful and patient during assembly, especially toward the end of construction. Once you’re done, you’ll have five good-looking solids that you can hang from your ceiling or display on your desk. Hold them in your hands and feel the symmetry.

The Platonic solids are described in Section 4.5, “The Platonic Solids Turn Amorous” on pages 270–275, with 3D pictures on pages 271–273. See also page 362 in Section 5.3, “Feeling Edgy?” To play with the Platonic solids virtually, check out your CD-ROM.

TOPOLOGY PUZZLE (also known as The Perplexing Pencil Puzzle)—This puzzle was invented by Sam Lloyd, a famous puzzle maker. It consists of a pencil, a metal cap, and a 14-inch piece of thin string that will allow you to “buttonhole a friend.” Attach the pencil to the buttonhole of your shirt. The challenge is to remove the pencil without cutting the string. It’s trickier than you may think.

Constructing the Puzzle:
- Seal the ends of a 14-inch length of thin string so the ends will not unravel.
- Double up the string, and tie the ends together with a knot to form a loop.
- Place the looped string inside the cap, and thread it through the hole until it is stopped by the knot. This step is almost as hard as the puzzle itself. To get the loop through that small hole, unfold a paper clip or find a 4-inch plastic-coated twister wire (found around electrical cords or a bag of bread). Strip away the plastic coating, and pass the wire through the hole from the outside of the cap. Loop the string over the wire and bend the wire to make a hook. Now, slowly pull the wire, and thus the string, through the hole in the cap. Be careful.
- Push the pencil all the way into the cap. The cap will fit best on the noneraser side of the pencil once the string has been pushed through the hole. (Optional: You might want to apply two drops of wood glue inside the cap before placing it over the pencil.)

Solving the Puzzle: Your first challenge is to attach the pencil to a buttonhole, as shown.

Your next challenge is to remove the pencil from the buttonhole. We suggest that you practice this on your own shirt or jacket before trying it on a “victim.” This puzzle is mentioned in Section 5.1, “Rubber Sheet Geometry,” Mindscape II.8 (page 339). Good luck! (Hint: Think about moving your shirt as well as the string and pencil.)

COOL DICE—The four peculiarly spotted dice in your kit provide a surprising paradox in probability and voting. These “cool dice” are first mentioned on pages 11–12 in Section 1.1, “Silly Stories Each with a Moral.” The applications of the dice are explained in Section 7.2, “Predicting the Future in an Uncertain World,” and Section 8.4, “Peril at the Polls.”

Your challenge is to order the dice so that if someone picks any of the four dice, you can always pick another one, from the remaining three, so that if you both roll your dice, the chances that your die will show a higher number than the other person’s is greater than half. In other words, no matter which die your friend selects, there will always be a die from the remaining three that will—on average—beat your friend’s more often than not! Try to find the right ordering and then challenge a friend. (See page 11, Mindscape IV.36 on page 539, and page 689.)