Chapter 3
Measures of Central Tendency and Variability

Advantages and Disadvantages of the Major Measures of Central Tendency

Advantages of the Mode:
1. Easy to find.
2. Can be used with any scale of measurement.
3. The only measure that can be used with a nominal scale.
4. Corresponds to actual score in the distribution.

Disadvantages of the Mode (the following apply when the mode is used with ordinal or interval/ratio data):
1. Generally unreliable, especially when representing a relatively small population (can change radically with only a minor change in the distribution).
2. Can be misleading; the mode tells you which score is most frequent, but tells you nothing about the other scores in the distribution (radical changes can be made to the distribution without changing the mode).
3. Cannot be easily used in conjunction with inferential statistics.

Advantages of the Median:
1. Can be used with either ordinal or interval/ratio data.
2. Can be used even if there are open-ended categories or undeterminable scores on either side of the distribution.
3. Provides a good representation of a typical score in a skewed distribution; is not unduly affected by extreme scores.
4. Minimizes the sum of the absolute deviations (i.e., the sum of score distances from the median -- ignoring sign -- is less than it would be from any other location in the distribution).

Disadvantages of the Median:
1. May not be an actual score in the distribution (e.g., if there are an even number of scores, or tied scores in the middle of the distribution).
2. Does not reflect the values of all the scores in the distribution (e.g., an extreme score can be moved even further out without affecting the median).
3. Compared to the mean, it is less reliable for drawing inferences about a population from a sample, and harder to use with advanced statistics.

Advantages of the Mean:
1. Reflects the values of all the scores in the distribution.
2. Has many desirable statistical properties.
3. Is the most reliable for drawing inferences, and the easiest to use in advanced statistical techniques.

Disadvantages of the Mean:
1. Usually not an actual score in the distribution.
2. Not appropriate for use with ordinal data.
3. Can be misleading when used to describe a skewed distribution.
4. Can be strongly affected by even just one very extreme score (i.e., an outlier).
Advantages and Disadvantages of the Major Measures of Variability

Advantages of the Range:
1. Easy to calculate.
2. Can be used with ordinal as well as interval/ratio data.
3. Encompasses entire distribution.

Disadvantages of the Range:
1. Depends on only two scores in the distribution and is therefore not reliable.
2. Cannot be found if there are undeterminable or open-ended scores at either end of the distribution.
3. Plays no role in advanced statistics.

Advantages of the SIQ Range:
1. Can be used with ordinal as well as interval/ratio data.
2. Can be found even if there are undeterminable or open-ended scores at either end of the distribution.
3. Not affected by extreme scores or outliers.

Disadvantages of the SIQ Range:
1. Does not take into account all the scores in the distribution.
2. Does not play a role in advanced statistical procedures.

Advantages of the Mean Deviation
1. Easy to understand (it is just the average distance from the mean).
2. Provides a good description of variability.
3. Takes into account all scores in the distribution.
4. Less sensitive to extreme scores than the standard deviation.

Disadvantages of the Mean Deviation
1. This measure is smaller when taken around the median than the mean.
2. Is not easily used in advanced statistics.
3. Cannot be calculated with undeterminable or open-ended scores.

Advantages of the Standard Deviation
1. Takes into account all scores in the distribution.
2. Provides a good description of variability.
3. Tends to be the most reliable measure.
4. Plays an important role in advanced statistical procedures.

Disadvantages of the Standard Deviation
1. Very sensitive to extreme scores or outliers.
2. Cannot be calculated with undeterminable or open-ended scores.

In order to illustrate the calculation of the various measures of variability, we will present hypothetical data for the following situation. You are a ninth-grade English teacher, and during the first class of the Fall term, you ask each of your 12 pupils how many books he or she has read over the summer vacation, to give you some idea of their interest in reading. The responses were as follows:
3, 1, 3, 3, 6, 2, 1, 7, 3, 4, 9, 2.

Putting the scores in order will help in finding some of the measures of variability:
1, 1, 2, 2, 3, 3, 3, 3, 4, 6, 7, 9.
The Range. The range is the highest score minus the lowest. The number of books read ranged from 1 to 9, so range = 9 - 1 = 8. If the scale is considered continuous (e.g., 9 books is really anywhere between 8 1/2 and 9 1/2 books), then range = upper real limit of highest score minus lower real limit of the lowest score = 9.5 - 0.5 = 9.

Interquartile Range. With such a small set of scores, grouping does not seem necessary to find the quartiles. Because N = 12, the 25th percentile is between the third and fourth scores. In this case, the third and fourth scores are both 2, so Q₁ = 2. Similarly, the 75th percentile is between the ninth and tenth score, which are 4 and 6, so Q₃ = 5. The IQ range = Q₃ - Q₁ = 5-2 = 3.

Semi-interquartile Range. The SIQ range = (Q₃ - Q₁)/2, which is half of the IQ range. For this example, SIQ range = 3/2 = 1.5.

Mean Deviation. This is the average of the absolute deviations from the mean. The mean for this example is μ = ΣX/N = 3.67. The mean deviation is found by applying Formula 1.2 to the data:

\[ M.D. = \frac{\sum |X_i - \mu|}{N} = \frac{1}{N} \left[ | -2.67| + | -2.67| + | -1.67| + | -1.67| + | .67| \right. \]
\[ \left. + | .67| + | .67| + | .33| + | 2.33| + | 3.33| + | 5.33| \right] = \frac{1}{12} (23.66) = 1.97 \]

Sum of Squares. This is the sum of the squared deviations from the mean, as found by the numerator of Formula 3.3:

\[ SS = \sum (X_i - \mu)^2 = (-2.67)^2 + (-2.67)^2 + (-1.67)^2 + ... + (2.33)^2 + (3.33)^2 + (5.33)^2 = \]
\[ = 7.13 + 7.13 + 2.79 + 2.79 + .45 + .45 + .45 + .45 + .11 + 5.44 + 11.11 + 28.44 = 66.75 \]

Variance. The average of the squared deviations from the mean, as found by Formula 3.4A:

\[ \sigma^2 = \frac{\sum (X_i - \mu)^2}{N} = \frac{SS}{N} = \frac{66.75}{12} = 5.56. \]

Standard Deviation. This equals the square-root of the variance as given by Formula 3.4B. For this example, \( \sigma = \sqrt{5.56} = 2.36. \)

Unbiased Sample Variance. If the 12 students in the English class were considered a sample of all ninth graders, and you wished to extrapolate from the sample to the population, you would use Formula 3.6A to calculate the unbiased variance s²:
Unbiased Standard Deviation. This equals the square-root of the unbiased variance as given by Formula 3.6B. For this example, \( s = \sqrt{6.068} = 2.46 \).

**Definitions of Key Terms**

**Central Tendency:** The location in a distribution that is most typical or best represents the entire distribution.

**Arithmetic mean:** This is the value obtained by summing all the scores in a group (or distribution), and then dividing by the number of scores summed. It is the most familiar measure of central tendency, and is therefore referred to simply as "the mean" or the "average."

**Mode:** The most frequent category, ordinal position, or score in a population or sample. **Unimodal:** describing a distribution with only one major peak (e.g., the normal distribution is unimodal). **Bimodal:** describing a distribution that has two roughly equal peaks (this shape usually indicates the presence of two distinct subgroups).

**Median:** This is the location in the distribution that is at the 50th percentile; half the scores are higher in value than the median while the other half are lower.

**Skewed distribution:** A distribution with the bulk of the scores closer to one end than the other, and relatively few scores in the other direction. **Positively skewed:** describing a distribution with a relatively small number of scores much higher than the majority of scores, but no scores much lower than the majority. **Negatively skewed:** opposite of positively-skewed (a few scores much lower than the majority, but none much higher).

**Open-ended category:** This is a measurement that has no limit on one end (e.g., 10 or more).

**Undeterminable score:** A score whose value has not been measured precisely but is usually known to be above or below some limit (e.g., subject has three minutes in which to answer a question, but does not answer at all).

**Box-and-whisker plot:** Also called boxplot, for short, this is a technique for exploratory data analysis devised by Tukey (1977). One can see the spread and symmetry of a distribution at a glance, and the position of any extreme scores.

**Hinges:** These are the sides of the box in a boxplot, corresponding approximately to the 25th and 75th percentiles of the distribution.

**H-spread:** The distance between the two hinges (i.e., the width of the box) in a boxplot.

**Inner fences:** Locations on either side of the box (in a boxplot) that are 1.5 times the H-spread from each hinge. The distance between the upper and lower inner fence is four times the H-spread.
Adjacent values: The *upper* adjacent value is the highest score in the distribution that is not higher than the upper inner fence, and the *lower* adjacent value is similarly defined in terms of the lower inner fence of a boxplot. The upper whisker is drawn from the upper hinge to the upper adjacent value, and the lower whisker is drawn from the lower hinge to the lower adjacent value.

Outlier: Defined in general as an extreme score standing by itself in a distribution, an outlier is more specifically defined in the context of a boxplot. In terms of a boxplot, an outlier is any score that is beyond the reach of the whiskers on either side. The outliers are indicated as points in the boxplot.

Range: The total width of the distribution as measured by subtracting the lowest score (or its lower real limit) from the highest score (or its upper real limit).

Interquartile (IQ) Range: The width of the middle half of the distribution as measured by subtracting the 25th percentile (Q1) from the 75th percentile (Q3).

Semi-interquartile (SIQ) Range: Half of the IQ range -- roughly half of the scores in the distribution are within this distance from the median.

Deviation Score: The difference between a score and a particular point in the distribution. When I refer to deviation scores, I will always mean the difference between a score and the mean (i.e., $X_i - \mu$).

Absolute Value: The magnitude of a number, ignoring its sign; that is, negative signs are dropped, and plus signs are left as is. *Absolute deviation scores* are the absolute values of deviation scores.

Mean Deviation: The mean of the absolute deviations from the mean of a distribution.

Sum of Squares (SS): The sum of the squared deviations from the mean of a distribution.

Population Variance ($\sigma^2$): Also called the mean-square (MS), this is the mean of the squared deviations from the mean of a distribution.

Population Standard Deviation ($\sigma$): Sometimes referred to as the root-mean-square (RMS) of the deviation scores, it is the square-root of the population variance.

Unbiased Sample Variance ($s^2$): This formula (i.e., $SS/N-1$), applied to a sample, provides an unbiased estimate of $\sigma^2$. When I refer in the text to just the "sample variance" (or use the symbol "$s^2$"), it is this formula to which I am referring.

Unbiased Sample Standard Deviation ($s$): The square-root of the unbiased sample variance. Although not a perfectly unbiased estimate of $\sigma$, I will refer to $s$ (i.e., $\sqrt{s^2}$) as the unbiased standard deviation of a sample, or just the sample standard deviation.

Degrees of Freedom (df): The number of scores that are free to vary after one or more parameters have been found for a distribution. When finding the variance or standard deviation of one sample, df = N-1, because the deviations are taken from the mean, which has already been found.

Definitional Formula: A formula that provides a clear definition of a sample statistic or population parameter, but may not be convenient for computational purposes. In the case of the variance or standard deviation, the definitional formula is also called the *deviational formula*, because it is based on finding deviation scores from the mean.
Computational Formula: An algebraic transformation of a definitional formula, which yields exactly the same value (except for any error due to intermediate rounding off), but reduces the amount or difficulty of the calculations involved.

Kurtosis: The degree to which a distribution bends sharply from the central peak to the tails, or slopes more gently, as compared to the normal distribution.

Leptokurtic: Refers to a distribution whose tails are relatively fatter than in the normal distribution, with a sharper peak in the middle; it has a positive value for kurtosis (given that the kurtosis of the normal distribution is set to zero).

Platykurtic: Refers to a distribution whose tails are relatively thinner than in the normal distribution, with a sharper peak in the middle; it has a negative value for kurtosis.

Mesokurtic: Refers to a distribution in which the thickness of the tails, and the sharpness of the central peak, is comparable to the normal distribution.

Practice Exercises
For this exercise, you will be using the dataset of body temperatures from the exercise in the previous chapter. The measurements are repeated here for your convenience:
97.6, 98.7, 96.9, 99.0, 93.2, 97.1, 98.5, 97.8, 94.5, 90.8, 99.7, 96.6, 97.8, 94.3, 91.7, 98.2, 95.3, 97.9, 99.6, 89.5, 93.0, 96.4, 94.8, 95.7, 97.4.

1. What is the mode, median, and mean for the data above? Which way does the distribution seem to be skewed?

2. What is the range, mean deviation, and standard deviation for these data?