CHAPTER 2

Fluid Properties

SIGNIFICANT LEARNING OUTCOMES

Conceptual Knowledge

- Define density, specific gravity, viscosity, surface tension, vapor pressure, and bulk modulus of elasticity.
- Describe the differences between absolute viscosity and kinematic viscosity.
- Describe how shear stress, viscosity, and the velocity distribution are related.
- Describe how viscosity, density, and vapor pressure vary with temperature and/or pressure.

Procedural Knowledge

- Look up fluid property values from figures, tables; know when and how to interpolate.
- Calculate gas density using the ideal gas law.

A fluid has certain characteristics by which its physical condition may be described. These characteristics are called properties of the fluid. This chapter introduces material properties of fluids and presents key equations, tables, and figures.

2.1 Properties Involving Mass and Weight

Mass and weight properties are needed for most problems in fluid mechanics, including the flow of ground water in aquifers and the pressure acting on a scuba diver or an underwater structure.

Mass Density, \( \rho \)

Mass density is defined as the ratio of mass to volume at a point, given by

\[
\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V}
\]  

(2.1)

Review the continuum assumption developed in Section 1.2 for the meaning of \( \Delta V \to 0 \). Mass density has units of kilograms per cubic meter (kg/m\(^3\)) or pounds-mass per cubic foot (lbm/ft\(^3\)). The mass density of water at 4°C is 1000 kg/m\(^3\) (62.4 lbm/ft\(^3\)), and it decreases slightly with increasing temperature, as shown in Table A.5. The mass density of air at 20°C and standard atmospheric pressure is 1.2 kg/m\(^3\) (0.075 lbm/ft\(^3\)), and it changes significantly with temperature and pressure. Mass density, often simply called density, is represented by the Greek symbol \( \rho \) (rho). The densities of common fluids are given in Tables A.2 to A.5.
**Specific Weight, \( \gamma \)**
The gravitational force per unit volume of fluid, or simply the weight per unit volume, is defined as *specific weight*. It is given the Greek symbol \( \gamma \) (gamma). Water at 20°C has a specific weight of 9790 N/m\(^3\) (or 62.4 lbf/ft\(^3\) at 50°F). In contrast, the specific weight of air at 20°C and standard atmospheric pressure is 11.8 N/m\(^3\). Specific weight and density are related by

\[
\gamma = \rho g
\]  
(2.2)

Specific weights of common liquids are given in Table A.4.

**Variation in Liquid Density**
In practice, engineers need to decide whether or not to model a fluid as constant density or variable density. Usually, a liquid such as water requires a large change in pressure in order to change the density. Thus, for most applications, liquids can be considered incompressible and can be assumed to have constant density. An exception to this occurs when different solutions, such as saline and fresh water, are mixed. A mixture of salt in water changes the density of the water without changing its volume. Therefore in some flows, such as in estuaries, density variations may occur within the flow field even though the fluid is essentially incompressible. A fluid wherein density varies spatially is described as *nonhomogeneous*. This text emphasizes the flow of *homogeneous* fluids, so the term *incompressible*, used throughout the text, implies constant density.

**Specific Gravity, \( S \)**
The ratio of the specific weight of a given fluid to the specific weight of water at the standard reference temperature 4°C is defined as *specific gravity*, \( S \):

\[
S = \frac{\gamma_{\text{fluid}}}{\gamma_{\text{water}}} = \frac{\rho_{\text{fluid}}}{\rho_{\text{water}}}
\]  
(2.3)

The specific weight of water at atmospheric pressure is 9790 N/m\(^3\). The specific gravity of mercury at 20°C is

\[
S_{\text{Hg}} = \frac{133 \text{ kN/m}^3}{9.79 \text{ kN/m}^3} = 13.6
\]

Because specific gravity is a ratio of specific weights, it is dimensionless and therefore independent of the system of units used.

### 2.2 Ideal Gas Law

The *ideal gas law* relates important thermodynamic properties, and is often used to calculate density.

One form of the law is

\[
pV = nR_uT
\]  
(2.4)

where \( p \) is the absolute pressure, \( V \) is the volume, \( n \) is the number of moles, \( R_u \) is the universal gas constant (the same for all gases), and \( T \) is absolute temperature. Absolute pressure,
EXAMPLE 2.1 DENSITY OF AIR
Air at standard sea-level pressure \( (p = 101 \text{kN/m}^2) \) has a temperature of 4°C. What is the density of the air?

**Problem Definition**

**Situation:** Air with a known temperature and pressure.

**Find:** Density \((\text{kg/m}^3)\).

**Properties:** Air, 4°C, \( p \) at 101 kN/m\(^2\); Table A.2, \( R = 287 \text{ J/kg K} \).

**Plan**

Apply the ideal gas law, Eq. (2.5), to solve for density, \( \rho \).

**Solution**

\[
\rho = \frac{p}{RT}
\]

\[
\rho = \frac{101 \times 10^3 \text{ N/m}^2}{287 \text{ J/kg K} \times (273 + 4) \text{ K}} = 1.27 \text{ kg/m}^3
\]

**Review**

1. Remember: Use absolute temperatures and pressures with the ideal gas law.
2. Remember: In Eq. (2.5), use \( R \) from Table A.2. Do not use \( R_u \).

introduced in Chapter 3, is referred to absolute zero. The universal gas constant is 8.314 kJ/kmol-K in the SI system and 1545 ft-lbf/lbmol°R in the traditional system. Eq. (2.4) can be rewritten as

\[
p = \frac{n M R_u}{\mathcal{V} M}
\]

where \( M \) is the molecular weight of the gas. The product of the number of moles and the molecular weight is the mass of the gas. Thus \( nM/\mathcal{V} \) is the mass per unit volume, or density. The quotient \( R_u/M \) is the gas constant, \( R \). Thus a second form of the ideal gas law is

\[
p = \rho RT
\]

(2.5)

Although no gas is ideal, Eq. (2.5) is a valid approximation for most gas flow problems. Values of \( R \) for a number of gases are given in Table A.2. To determine the mass density of a gas, solve Eq. (2.5) for \( \rho \):

\[
\rho = \frac{p}{RT}
\]

Properties Involving Thermal Energy

**Specific Heat, \( c \)**

The property that describes the capacity of a substance to store thermal energy is called **specific heat**. By definition, it is the amount of thermal energy that must be transferred to a unit mass of substance to raise its temperature by one degree. The specific heat of a gas depends on the process accompanying the change in temperature. If the **specific volume** \( \nu \) of the gas (\( \nu = 1/\rho \)) remains constant while the temperature changes, then the specific heat is identified as \( c_v \). However, if the pressure is held constant during the change in state, then the specific heat is identified as \( c_p \). The ratio \( c_p/c_v \) is given the symbol \( k \). Values for \( c_p \) and \( k \) for various gases are given in Table A.2.
Internal Energy
The energy that a substance possesses because of the state of the molecular activity in the substance is termed *internal energy*. Internal energy is usually expressed as a specific quantity—that is, internal energy per unit mass. In the SI system, the specific internal energy, $u$, is given in joules per kilogram; in Traditional Units it is given in Btu/ibm. The internal energy is generally a function of temperature and pressure. However, for an ideal gas, it is a function of temperature alone.

Enthalpy
The combination $u + p/\rho$ is encountered frequently in equations for thermodynamics and compressible flow; it has been given the name *specific enthalpy*. For an ideal gas, $u$ and $p/\rho$ are functions of temperature alone. Consequently their sum, specific enthalpy, is also a function solely of temperature.

Viscosity
The property of viscosity is important to engineering practice because it leads to significant energy loss when moving fluids contact a solid boundary, or when different zones of fluid are flowing at different velocities.

Viscosity, $\mu$
*Viscosity* (also called *dynamic viscosity*, or *absolute viscosity*) is a measure of a fluid’s resistance to deformation under shear stress. For example, crude oil has a higher resistance to shear than does water. Crude oil will pour more slowly than water from an identical beaker held at the same angle. This relative slowness of the oil implies a low “speed” or rate of strain. The symbol used to represent viscosity is $\mu$ (mu). To understand the physics of viscosity, it is useful to refer back to solid mechanics and the concepts of shear stress and shear strain. Shear stress, $\tau$, tau, is the ratio of force/area on a surface when the force is aligned parallel to the area. Shear strain is a change in an interior angle of a cubical element, $\Delta\phi$, that was originally a right angle. The shear stress on a material element in solid mechanics is proportional to the strain, and the constant of proportionality is the shear modulus:

\[
\{\text{shear stress}\} = \{\text{shear modulus}\} \times \{\text{strain}\}
\]

In fluid flow, however, the shear stress on a fluid element is proportional to the rate (speed) of strain, and the constant of proportionality is the viscosity:

\[
\{\text{shear stress}\} = \{\text{viscosity}\} \times \{\text{rate of strain}\}
\]

Figure 2.1 depicts an initially rectangular element in a parallel flow field. As the element moves downstream, a shear force on the top of the element (and a corresponding shear stress in the opposite direction on the bottom of the element) causes the top surface to move faster (with velocity $V + \Delta V$) than the bottom (at velocity $V$). The forward and rearward edges become inclined at an angle $\Delta\phi$ with respect to the vertical. The rate at which $\Delta\phi$ changes with time, given by $\dot{\phi}$, is the *rate of strain*, and can be related to the velocity difference between...
the two surfaces. In time ($\Delta t$) the upper surface moves $(V + \Delta V)\Delta t$ while the bottom surface moves $V\Delta t$, so the net difference is $\Delta V\Delta t$. The strain $\Delta \phi$ is

$$\Delta \phi = \frac{\Delta V\Delta t}{\Delta y}$$

where $\Delta y$ is the distance between the two surfaces. The rate of strain is

$$\frac{\Delta \phi}{\Delta t} = \frac{\Delta V}{\Delta y}$$

In the limit as $\Delta t \to 0$ and $\Delta y \to 0$, the rate of strain is related to the velocity gradient by $\phi = dV/dy$, so the shear stress (shear force per unit area) is

$$\tau = \mu \frac{dV}{dy} \quad (2.6)$$

For strain in flow near a wall, as shown in Fig. 2.2, the term $dV/dy$ represents the velocity gradient (or change of velocity with distance from the wall), where $V$ is the fluid velocity and $y$ is the distance measured from the wall. The velocity distribution shown is characteristic of flow next to a stationary solid boundary, such as fluid flowing through a pipe. Several observations relating to this figure will help one to appreciate the interaction between viscosity and velocity distribution. First, the velocity gradient at the boundary is finite. The curve of velocity variation cannot be tangent to the boundary because this would imply an infinite velocity gradient and, in turn, an infinite shear stress, which is impossible. Second, a velocity gradient that becomes less steep ($dV/\Delta y$ becomes smaller) with distance from the boundary has a maximum shear stress at the boundary, and the shear stress decreases with distance from the boundary. Also note that the velocity of the fluid is zero at the stationary boundary. That is, at the boundary surface the fluid has the velocity of the boundary—
no slip occurs between the fluid and the boundary. This is referred to as the no-slip condition. The no-slip condition is characteristic of all flows used in this text.

From Eq. (2.6) it can be seen that the viscosity \( \mu \) is related to the shear stress and velocity gradient.

\[
\mu = \frac{\tau}{dV/dy} = \frac{N/m^2}{(m/s)/m} = N \cdot s/m^2
\]

A common unit of viscosity is the poise, which is 1 dyne-s/cm² or 0.1 N · s/m². The viscosity of water at 20°C is one centipoise (10⁻² poise) or 10⁻³ N · s/m². The unit of viscosity in the traditional system is lb·s/ft².

**Kinematic Viscosity, \( \nu \)**

Many equations of fluid mechanics include the ratio \( \mu/\rho \). Because it occurs so frequently, this ratio has been given the special name kinematic viscosity. The symbol used to identify kinematic viscosity is \( \nu \) (nu). Units of kinematic viscosity \( \nu \) are m²/s, as shown.

\[
\nu = \frac{\mu}{\rho} \Rightarrow \frac{N \cdot s/m^2}{kg/m^3} = m^2/s
\]

The units for kinematic viscosity in the traditional system are ft²/s.

**Temperature Dependency**

The effect of temperature on viscosity is different for liquids and gases. The viscosity of liquids decreases as the temperature increases, whereas the viscosity of gases increases with increasing temperature; this trend is also true for kinematic viscosity (see Fig. 2.3 and Figs. A.2 and A.3).

To understand the mechanisms responsible for an increase in temperature that causes a decrease in viscosity in a liquid, it is helpful to rely on an approximate theory that has been developed to explain the observed trends (1). The molecules in a liquid form a lattice-like structure with “holes” where there are no molecules, as shown in Fig. 2.4. Even when the liquid is at rest, the molecules are in constant motion, but confined to cells, or “cages.” The cage or lattice structure is caused by attractive forces between the molecules. The cages may be thought of as energy barriers. When the liquid is subjected to a rate of strain and thus caused to move, as shown in Fig. 2.4, there is a shear stress, \( \tau \), imposed by one layer on another in the fluid. This force/area assists a molecule in overcoming the energy barrier, and it can move into the next hole. The magnitude of these energy barriers is related to viscosity, or resistance.
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To shear deformation. At a higher temperature the size of the energy barrier is smaller, and it is easier for molecules to make the jump, so that the net effect is less resistance to deformation under shear. Thus, an increase in temperature causes a decrease in viscosity for liquids.

An equation for the variation of liquid viscosity with temperature is

\[ \mu = Ce^{b/T} \]  \hspace{1cm} (2.9)

where \( C \) and \( b \) are empirical constants that require viscosity data at two temperatures for evaluation. This equation should be used primarily for data interpolation. The variation of viscosity (dynamic and kinematic) for other fluids is given in Figs. A.2 and A.3.

As compared to liquids, gases do not have zones or cages to which molecules are confined by intermolecular bonding. Gas molecules are always undergoing random motion. If this random motion of molecules is superimposed upon two layers of gas, where the top layer is moving faster than the bottom layer, periodically a gas molecule will randomly move from one layer to the other. This behavior of a molecule in a low-density gas is analogous to people jumping back and forth between two conveyor belts moving at different speeds as shown in Fig. 2.5. When people jump from the high-speed belt to the low-speed belt, a re-
straining (or braking) force has to be applied to slow the person down (analogous to viscosity). If the people are heavier, or are moving faster, a greater braking force must be applied. This analogy also applies for gas molecules translating between fluid layers where a shear force is needed to maintain the layer speeds. As the gas temperature increases, more of the molecules will be making random jumps. Just as the jumping person causes a braking action on the belt, highly mobile gas molecules have momentum, which must be resisted by the layer to which the molecules jump. Therefore, as the temperature increases, the viscosity, or resistance to shear, also increases.

**Figure 2.5**

Analogy of people moving between conveyor belts and gas molecules translating between fluid layers.

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**EXAMPLE 2.2 CALCULATING VISCOSITY OF LIQUID AS A FUNCTION OF TEMPERATURE**

The dynamic viscosity of water at 20°C is $1.00 \times 10^{-3} \text{ N} \cdot \text{s} / \text{m}^2$, and the viscosity at 40°C is $6.53 \times 10^{-4} \text{ N} \cdot \text{s} / \text{m}^2$. Using Eq. (2.9), estimate the viscosity at 30°C.

**Problem Definition**

**Situation:** Viscosity of water is specified at two temperatures.

**Find:** The viscosity at 30°C by interpolation.

**Properties:**

a) Water at 20°C, $\mu = 1.00 \times 10^{-3} \text{ N} \cdot \text{s} / \text{m}^2$.

b) Water at 40°C, $\mu = 6.53 \times 10^{-4} \text{ N} \cdot \text{s} / \text{m}^2$.

**Plan**

1. Linearize Eq. (2.9) by taking the logarithm.
2. Interpolate between the two known values of viscosity.
3. Solve for $\ln C$ and $b$ in this linear set of equations.
4. Change back to exponential equation, and solve for $\mu$ at 30°C.

**Solution**

1. Logarithm of Eq. (2.9)

   $$\ln \mu = \ln C + \frac{b}{T}$$

2. Interpolation

   $$-6.908 = \ln C + 0.00341b$$

   $$-7.334 = \ln C + 0.00319b$$

3. Solution for $\ln C$ and $b$

   $$\ln C = -13.51 \quad b = 1936 \text{ (K)}$$

   $$C = e^{-13.51} = 1.357 \times 10^{-6}$$

4. Substitution back in exponential equation

   $$\mu = 1.357 \times 10^{-6} e^{1936/T}$$

   At 30°C

   $$\mu = 8.08 \times 10^{-4} \text{ N} \cdot \text{s} / \text{m}^2$$

**Review**

Note: This value differs by 1% from the reported value in Table A.5, but provides a much better estimate than would be obtained by arithmetically averaging two values on the table.
EXAMPLE 2.3 MODELING A BOARD SLIDING ON A LIQUID LAYER

A board 1 m by 1 m that weighs 25 N slides down an inclined ramp (slope = 20°) with a velocity of 2.0 cm/s. The board is separated from the ramp by a thin film of oil with a viscosity of 0.05 N · s/m². Neglecting edge effects, calculate the space between the board and the ramp.

Problem Definition

Situation: A board is sliding down a ramp, on a thin film of oil.
Find: Space (in m) between the board and the ramp.
Assumptions: A linear velocity distribution in the oil.
Properties: Oil, μ = 0.05 N · s/m².
Sketch:

Plan

1. Draw a free body diagram of the board, as shown in “sketch.”
   • For a constant sliding velocity, the resisting shear force is equal to the component of weight parallel to the inclined ramp.
   • Relate shear force to viscosity and velocity distribution.

Solution

1. Freebody analysis

\[ F_{\text{tangential}} = F_{\text{shear}} \]
\[ W \sin 20° = \tau A \]
\[ W \sin 20° = \mu \frac{dV}{dy} A \]

2. Substitution of \( dV/\Delta y \) as \( \Delta V/\Delta y \)

\[ W \sin 20° = \mu \frac{\Delta V}{\Delta y} A \]

3. Solution for \( \Delta y \)

\[ \Delta y = \frac{\mu \Delta V A}{W \sin 20°} \]
\[ \Delta y = \frac{0.05 \text{ N} \cdot \text{s/m}^2 \times 0.020 \text{ m/s} \times 1 \text{ m}^2}{25 \text{ N} \times \sin 20°} \]
\[ \Delta y = 0.000117 \text{ m} \]
\[ \Delta y = 0.117 \text{ mm} \]

An estimate for the variation of gas viscosity with temperature is Sutherland’s equation,

\[
\frac{\mu}{\mu_0} = \left( \frac{T}{T_0} \right)^{3/2} \frac{T_0 + S}{T + S}
\]  

(2.10)

where \( \mu_0 \) is the viscosity at temperature \( T_0 \), and \( S \) is Sutherland’s constant. All temperatures are absolute. Sutherland’s constant for air is 111 K; values for other gases are given in Table A.2. Using Sutherland’s equation for air yields viscosities with an accuracy of ±2% for temperatures between 170 K and 1900 K. In general, the effect of pressure on the viscosity of common gases is minimal for pressures less than 10 atmospheres.

Newtonian versus Non-Newtonian Fluids

Fluids for which the shear stress is directly proportional to the rate of strain are called Newtonian fluids. Because shear stress is directly proportional to the shear strain, \( dV/dy \), a plot relating these variables (see Fig. 2.6) results in a straight line passing through the origin. The slope of this line is the value of the dynamic (absolute) viscosity. For some fluids the shear...
stress may not be directly proportional to the rate of strain; these are called non-Newtonian fluids. One class of non-Newtonian fluids, shear-thinning fluids, has the interesting property that the ratio of shear stress to shear strain decreases as the shear strain increases (see Fig. 2.6). Some common shear-thinning fluids are toothpaste, catsup, paints, and printer’s ink. Fluids for which the viscosity increases with shear rate are shear-thickening fluids. Some examples of these fluids are mixtures of glass particles in water and gypsum-water mixtures. Another type of non-Newtonian fluid, called a Bingham plastic, acts like a solid for small values of shear stress and then behaves as a fluid at higher shear stress. The shear stress versus shear strain rate for a Bingham plastic is also shown in Fig. 2.6. This book will focus on the theory and applications involving Newtonian fluids. For more information on the theory of flow of non-Newtonian fluids, see references (2) and (3).

2.5 Bulk Modulus of Elasticity

The bulk modulus of elasticity, \( E_v \), is a property that relates changes in pressure to changes in volume (e.g., expansion or contraction)

\[
E_v = - \frac{dp}{d \mathcal{V}/\mathcal{V}} = - \frac{\text{change in pressure}}{\text{fractional change in volume}} \quad (2.11)
\]

where \( dp \) is the differential pressure change, \( d\mathcal{V} \) is the differential volume change, and \( \mathcal{V} \) is the volume of fluid. Because \( d\mathcal{V}/\mathcal{V} \) is negative for a positive \( dp \), a negative sign is used in the definition to yield a positive \( E_v \). The elasticity is often called the compressibility of the fluid.

The fractional change in volume can be related to the change in material density using

\[
M = \rho \mathcal{V} \quad (2.12)
\]

Since the mass is constant

\[
dM = \rho \, d\mathcal{V} + \mathcal{V} \, dp = 0
\]
2.6 SURFACE TENSION

so

\[ \nabla d\rho = -\rho \ nabla \ \text{or} \ \frac{d\rho}{\rho} = -\frac{dV}{V} \]

and the definition of the bulk modulus of elasticity becomes

\[ E_v = \frac{d\rho/\rho}{dV} = \frac{\text{change in pressure}}{\text{fractional change in density}} \]  \hspace{1cm} (2.13)

The bulk modulus of elasticity of water is approximately 2.2 GN/m^2, which corresponds to a 0.05% change in volume for a change of 1 MN/m^2 in pressure. Obviously, the term *incompressible* is justifiably applied to water because it has such a small change in volume for a very large change in pressure.

The elasticity of an ideal gas is proportional to the pressure, according to the ideal gas law. For an isothermal (constant-temperature) process,

\[ \frac{dp}{dT} = RT \]

so

\[ E_v = \rho \frac{dp}{dT} = \rho RT = p \]

For an adiabatic process, \( E_v = k\rho \), where \( k \) is the ratio of specific heats, \( c_p/c_v \).

The elasticity or compressibility of a gas is important in high-speed gas flows where pressure variations can cause significant density changes. As will be shown in Chapter 12, the elasticity of a gas is related to the speed of sound in that gas. The ratio of the flow velocity to the speed of sound is the Mach number, which relates to the importance of elasticity effects.

Surface Tension

*Surface tension*, \( \sigma \) (sigma), is a material property whereby a liquid at a material interface, usually liquid-gas, exerts a force per unit length along the surface. According to the theory of molecular attraction, molecules of liquid considerably below the surface act on each other by forces that are equal in all directions. However, molecules near the surface have a greater attraction for each other than they do for molecules below the surface because of the presence of a different substance above the surface. This produces a layer of surface molecules on the liquid that acts like a stretched membrane. Because of this membrane effect, each portion of the liquid surface exerts “tension” on adjacent portions of the surface or on objects that are in contact with the liquid surface. This tension acts in the plane of the surface, and is given by:

\[ F_\sigma = \sigma L \]  \hspace{1cm} (2.14)

where \( L \) is the length over which the surface tension acts.

Surface tension for a water–air surface is 0.073 N/m at room temperature. The magnitude of surface tension decreases with increasing temperature; tabulated values for different liquids as a function of temperature are available in the literature and online. The effect of surface tension is illustrated for the case of *capillary action* (rise above a static water level at atmospheric pressure) in a small tube (Fig. 2.7). Here the end of a small-diameter tube is
inserted into a reservoir of water, and the characteristic curved water surface profile occurs within the tube. The relatively greater attraction of the water molecules for the glass rather than the air causes the water surface to curve upward in the region of the glass wall. Then the surface tension force acts around the circumference of the tube, in the direction indicated. It may be assumed that the contact angle $\theta$ is equal to 0° for water against glass. The surface tension force produces a net upward force on the water that causes the water in the tube to rise above the water surface in the reservoir. A calculation of the surface tension force acting to raise the water in a small-diameter tube is demonstrated in Example 2.4.

Other manifestations of surface tension include the excess pressure (over and above atmospheric pressure) created inside droplets and bubbles because there is necessarily a pressure difference across a curved interface; the breakup of a liquid jet into droplets; the shape and motion of bubbles, the structure of foams, and the binding together of wetted granular material, such as soil.

Surface tension forces for several different cases are shown in Fig. 2.8. Case (a) is a spherical droplet of radius $r$. The surface tension force is balanced by the internal pressure.

$$F_\sigma = \sigma L = pA$$

or

$$2\pi r \sigma = p \pi r^2$$

$$p = \frac{2\sigma}{r}$$

Case (b) is a bubble of radius $r$ that has internal and external surfaces and the surface-tension force acts on both surfaces, so

$$p = \frac{2\sigma}{r}$$

Case (c) is a cylinder supported by surface-tension forces. The liquid does not wet the cylinder surface. The maximum weight the surface tension can support is

$$W = 2F_\sigma = 2\sigma L$$

where $L$ is the length of the cylinder.

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**EXAMPLE 2.4  CAPILLARY RISE IN A TUBE**

To what height above the reservoir level will water (at 20°C) rise in a glass tube, such as that shown in Fig. 2.7, if the inside diameter of the tube is 1.6 mm?

**Problem Definition**

**Situation:** A glass tube of small diameter placed in an open reservoir of water induces capillary rise.

**Find:** The height the water will rise above the reservoir level.

**Sketch:** See Figure 2.7.

**Properties:** Water (20 °C), Table A.5, $\sigma = 0.073$ N/m; $\gamma = 9790$ N/m³.

**Plan**

1. Perform a force balance on water that has risen in the tube.
2. Solve for $\Delta h$.

**Solution**

1. Force balance: Weight of water (down) is balanced by surface tension force (up).

$$F_{\sigma, z} - W = 0$$

$$\sigma \pi d \cos \theta - \gamma (\Delta h) \left(\frac{\pi d^2}{4}\right) = 0$$

Because the contact angle $\theta$ for water against glass is so small, it can be assumed to be 0°; therefore $\cos \theta = 1$. Therefore:

$$\sigma \pi d - \gamma (\Delta h) \left(\frac{\pi d^2}{4}\right) = 0$$

2. Solve for $\Delta h$:

$$\Delta h = \frac{4\sigma}{\gamma d} = \frac{4 \times 0.073 \text{ N/m}}{9790 \text{ N/m}^3 \times 1.6 \times 10^{-3} \text{ m}} = 18.6 \text{ mm}$$
**2.7 VAPOR PRESSURE**

**Figure 2.8**

*Surface-tension forces for several different cases.*

(a) Spherical droplet

(b) Spherical bubble

(c) Cylinder supported by surface tension (liquid does not wet cylinder)

(d) Ring pulled out of liquid (liquid wets the ring)

Case (d) is a ring being pulled out of a liquid. This is a technique to measure surface tension. The force due to surface tension on the ring is

\[ F_\sigma = F_{\sigma,i} + F_{\sigma,o} \]

\[ = \pi \sigma (D_l + D_o) \]

### 2.7 Vapor Pressure

The pressure at which a liquid will vaporize, or boil, at a given temperature, is called its *vapor pressure*. This means that boiling occurs whenever the local pressure equals the vapor pressure. Vapor pressure increases with temperature. Note that there are two ways to boil a liquid. One way is to raise the temperature, assuming that the pressure is fixed. For water at 14.7 psia, this can be accomplished by increasing the temperature of water at sea level to 212°F, thus reaching the temperature where the vapor pressure is equal to the same value. However, boiling can also occur in water at temperatures much below 212°F if the pressure in the water is reduced to the vapor pressure of water corresponding to that lower temperature. For example, the vapor pressure of water at 50°F (10°C) is 0.178 psia (approximately 1% of standard atmospheric pressure). Therefore, if the pressure in water at 50°F is reduced to 0.178 psia, the water boils.*

Such boiling often occurs in localized low-pressure zones of flowing liquids, such as on the suction side of a pump. When localized low-pressure boiling does occur in flowing liquids, vapor bubbles start growing in local regions of very low pressure and then collapse in regions of higher pressure downstream. This phenomenon, which is called *cavitation*, can cause extensive damage to fluids systems, and is discussed in Chapter 5.

Table A.5 gives values of vapor pressure for water.

* Actually, boiling can occur at this vapor pressure only if there is a gas–liquid surface present to allow the process to start. Boiling at the bottom of a pot of water is usually initiated in crevices in the material of the pot, in which minute bubbles of air are entrapped even when the pot is filled with water.
**Summary**

The commonly used fluid properties are

- **Mass density** ($\rho$): mass per unit volume.
- **Specific weight** ($\gamma$): weight per unit volume.
- **Specific gravity** ($S$): ratio of specific weight to specific weight of water at reference conditions.

The relationship between pressure, density, and temperature for an ideal gas is

$$p = \rho RT$$

where $R$ is the gas constant, and pressure and temperature must be expressed in absolute values.

In a fluid the shear stress is proportional to the rate of strain, and the constant of proportionality is the viscosity. The shear stress at a wall is given by

$$\tau = \mu \frac{dV}{dy}$$

where $dV/dy$ is the velocity gradient of the fluid evaluated at the wall. In a Newtonian fluid, the viscosity is independent of the rate of strain. A fluid for which the effective viscosity decreases with increasing strain rate is a shear-thinning fluid.

Surface tension is the result of molecular attraction near a free surface, causing the surface to act like a stretched membrane.

The bulk modulus of elasticity relates to the pressure required to change the density of a fluid.

When the local pressure is equal to the vapor pressure at a given temperature, liquid boils.

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**References**


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**Problems**

*A Preview Question (PQ) can be assigned prior to in-class coverage of a topic.*

**Properties Related to Mass and Weight**

*2.1 PQ* Describe how density differs from specific weight.

*2.2 PQ* For what fluids can we (usually) assume density to be nearly constant? For what fluids must we be careful to calculate density as a function of temperature and pressure?

*2.3 PQ* Where in this text can you find density data for such fluids as oil and mercury?

*2.4* An engineer living at an elevation of 2500 ft is conducting experiments to verify predictions of glider performance. To process data, density of ambient air is needed. The engineer measures temperature (74.3°F) and atmospheric pressure (27.3 inches of mercury). Calculate density in units of kg/m³. Compare the calculated value with data from Table A.3 and make a recommendation about the effects of elevation on density; that is, are the effects of elevation significant?

*2.5* Calculate the density and specific weight of carbon dioxide at a pressure of 300 kN/m² absolute and 60°C.
PROBLEMS

2.6 Determine the density and specific weight of methane gas at a pressure of 300 kN/m² absolute and 60°C.

2.7 Natural gas is stored in a spherical tank at a temperature of 10°C. At a given initial time, the pressure in the tank is 100 kPa gage, and the atmospheric pressure is 100 kPa absolute. Some time later, after considerably more gas is pumped into the tank, the pressure in the tank is 200 kPa gage, and the temperature is still 10°C. What will be the ratio of the mass of natural gas in the tank when \( p = 200 \text{ kPa gage} \) to that when the pressure was 100 kPa gage?

2.8 At a temperature of 100°C and an absolute pressure of 5 atmospheres, what is the ratio of the density of water to the density of air, \( \rho_w/\rho_a \)?

2.9 Find the total weight of a 10 ft³ tank of oxygen if the oxygen is pressurized to 500 psia, the tank itself weighs 150 lbf, and the temperature is 70°F?

2.10 A 10 m³ oxygen tank is at 15°C and 800 kPa. The valve is opened, and some oxygen is released until the pressure in the tank drops to 600 kPa. Calculate the mass of oxygen that has been released from the tank if the temperature in the tank does not change during the process.

2.11 What are the specific weight and density of air at an absolute pressure of 600 kPa and a temperature of 50°C?

2.12 Meteorologists often refer to air masses in forecasting the weather. Estimate the mass of 1 m³ of air in slugs and kilograms. Make your own reasonable assumptions with respect to the conditions of the atmosphere.

2.13 A bicycle rider has several reasons to be interested in the effects of temperature on air density. The aerodynamic drag force decreases linearly with density. Also, a change in temperature will affect the tire pressure.

2.14 A design team is developing a prototype CO₂ cartridge for a manufacturer of rubber rafts. This cartridge will allow a user to quickly inflate a raft. A typical raft is shown in the sketch. Assume a raft inflation pressure of 3 psi (this means that the absolute pressure is 3 psi greater than local atmospheric pressure). Estimate the volume of the raft and the mass of CO₂ in grams. Make your own reasonable assumptions with respect to the prototype cartridge.

2.15 A team is designing a helium-filled balloon that will fly to an altitude of 80,000 ft. As the balloon ascends, the upward force (buoyant force) will need to exceed the total weight. Thus, weight is critical. Estimate the weight (in newtons) of the helium inside the balloon. The balloon is inflated at a site where the atmospheric pressure is 0.89 bar and the temperature is 22°C. When inflated prior to launch, the balloon is spherical (radius 1.3 m) and the inflation pressure equals the local atmospheric pressure.

2.16 Hydrometers are used in the wine and beer industries to measure the alcohol content of the product. This is accomplished by measuring the specific gravity of the liquid before fermentation, during fermentation, or after fermentation is complete. During fermentation, glucose \((C_6H_{12}O_6)\) is converted to ethyl alcohol \((CH_3CH_2OH)\) and carbon dioxide gas, which escapes from the vat.

\[
C_6H_{12}O_6 \rightarrow 2(CH_3CH_2OH) + 2(CO_2)
\]

Brewer's yeast tolerates alcohol contents to approximately 5% before fermentation stops, whereas wine yeast tolerates alcohol contents up to 21% depending on the yeast strain. The specific gravity of alcohol is 0.80, and the maximum specific gravity of sugar in solution is 1.59. If a wine has a specific gravity of 1.08 before fermentation, and all the sugar is converted to alcohol, what will be the final specific gravity of the wine and the percent alcohol content by volume? Assume that the initial liquid (the unfermented wine is called must) contains only sugar and water.

Viscosity

2.17 The following questions relate to viscosity.

a. What are the primary dimensions of viscosity? What are five common units?

b. What is the viscosity of SAE 10W-30 motor oil at 115°F (in traditional units)?

c. How does viscosity of water vary with temperature? Why?

d. How does viscosity of air vary with temperature? Why?

2.18 What is the change in the viscosity and density of water between 10°C and 70°C? What is the change in the viscosity and density of air between 10°C and 70°C? Assume standard atmospheric pressure \((p = 101 \text{ kN/m² absolute})\).

2.19 Determine the change in the kinematic viscosity of air that is heated from 10°C to 70°C. Assume standard atmospheric pressure.

2.20 Find the dynamic and kinematic viscosities of kerosene, SAE 10W-30 motor oil, and water at a temperature of 38°C (100°F).

2.21 What is the ratio of the dynamic viscosity of air to that of water at standard pressure and a temperature of 20°C? What is the ratio of the kinematic viscosity of air to that of water for the same conditions?
2.22 Using Sutherland’s equation and the ideal gas law, develop an expression for the kinematic viscosity ratio \( \frac{\nu}{\nu_0} \) in terms of pressures \( p \) and \( p_0 \) and temperatures \( T \) and \( T_0 \), where the subscript 0 refers to a reference condition.

2.23 The dynamic viscosity of air at 15°C is \( 1.78 \times 10^{-5} \) N \( \cdot \) s/m\(^2\). Using Sutherland’s equation, find the viscosity at 100°C.

2.24 The kinematic viscosity of methane at 15°C and atmospheric pressure is \( 1.59 \times 10^{-5} \) m\(^2\) /s. Using Sutherland’s equation and the ideal gas law, find the kinematic viscosity at 200°C and 2 atmospheres.

2.25 The dynamic viscosity of nitrogen at 59°F is \( 3.59 \times 10^{-7} \) lbf \( \cdot \) s/ft\(^2\). Using Sutherland’s equation, find the dynamic viscosity at 200°F.

2.26 The kinematic viscosity of helium at 59°F and 1 atmosphere is \( 1.22 \times 10^{-5} \) ft\(^2\)/s. Using Sutherland’s equation and the ideal gas law, find the kinematic viscosity at 30°F and a pressure of 1.5 atmospheres.

2.27 The absolute viscosity of propane at 100°C is \( 1.00 \times 10^{-5} \) N \( \cdot \) s/m\(^2\) and at 400°C is \( 1.72 \times 10^{-5} \) N \( \cdot \) s/m\(^2\). Find Sutherland’s constant for propane.

2.28 Ammonia is very volatile, so it may be either a gas or a liquid at room temperature. When it is a gas, its absolute viscosity at 68°F is \( 2.07 \times 10^{-7} \) lbf \( \cdot \) s/ft\(^2\) and at 392°F is \( 3.46 \times 10^{-7} \) lbf \( \cdot \) s/ft\(^2\). Using these two data points, find Sutherland’s constant for ammonia.

2.29 The viscosity of SAE 10W-30 motor oil at 38°C is 0.067 N \( \cdot \) s/m\(^2\) and at 99°C is 0.011 N \( \cdot \) s/m\(^2\). Using Eq. (2.8) for interpolation, find the viscosity at 60°C. Compare this value with that obtained by linear interpolation.

2.30 The viscosity of grade 100 aviation oil at 100°F is \( 4.43 \times 10^{-3} \) lbf \( \cdot \) s/ft\(^2\) and at 210°F is \( 3.9 \times 10^{-4} \) lbf \( \cdot \) s/ft\(^2\). Using Eq. (2.8), find the viscosity at 150°F.

2.31 Two plates are separated by a 1/8-in. space. The lower plate is stationary; the upper plate moves at a velocity of 25 ft/s. Oil (SAE 10W-30, 150°F), which fills the space between the plates, has the same velocity as the plates at the surface of contact. The variation in velocity of the oil is linear. What is the shear stress in the oil?

2.32 Find the kinematic and dynamic viscosities of air and water at a temperature of 40°C (104°F) and an absolute pressure of 170 kPa (25 psia).

2.33 The sliding plate viscometer shown below is used to measure the viscosity of a fluid. The top plate is moving to the right with a constant velocity of 10 m/s in response to a force of 3 N. The bottom plate is stationary. What is the viscosity of the fluid? Assume a linear velocity distribution.

2.34 The velocity distribution for water (20°C) near a wall is given by \( u = a(y/b)^{1/6} \), where \( a = 10 \) m/s, \( b = 2 \) mm, and \( y \) is the distance from the wall in mm. Determine the shear stress in the water at \( y = 1 \) mm.

2.35 The velocity distribution for the flow of crude oil at 100°F (μ = 8 \( \times \) 10\(^{-5} \) lbf \( \cdot \) s/ft\(^2\)) between two walls is shown, and is given by \( u = 100y(0.1 - y) \) ft/s, where \( y \) is measured in feet and the space between the walls is 0.1 ft. Plot the velocity distribution and determine the shear stress at the walls.

2.36 A liquid flows between parallel boundaries as shown above. The velocity distribution near the lower wall is given in the following table:

<table>
<thead>
<tr>
<th>y in mm</th>
<th>V in m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.00</td>
</tr>
<tr>
<td>1.0</td>
<td>1.00</td>
</tr>
<tr>
<td>2.0</td>
<td>1.99</td>
</tr>
<tr>
<td>3.0</td>
<td>2.98</td>
</tr>
</tbody>
</table>

a. If the viscosity of the liquid is \( 10^{-5} \) N \( \cdot \) s/m\(^2\), what is the maximum shear stress in the liquid?

b. Where will the minimum shear stress occur?

2.37 Suppose that glycerin is flowing \( (T = 20°C) \) and that the pressure gradient \( dp/ds \) is \(-1.6\) kN/m\(^3\). What are the velocity and shear stress at a distance of 12 mm from the wall if the space \( B \) between the walls is 5.0 cm? What are the shear stress and velocity at the wall? The velocity distribution for viscous flow between stationary plates is \( u = \frac{1}{2\mu} \frac{dp}{dx} (By - y^2) \).

2.38 A laminar flow occurs between two horizontal parallel plates under a pressure gradient \( dp/ds \) (\( p \) decreases in the positive \( s \) direction). The upper plate moves left (negative) at velocity \( u_L \). The expression for local velocity \( u \) is given as \( u = \frac{1}{2\mu} \frac{dp}{ds} (Hy - y^2) + u_L \frac{y}{H} \).

a. Is the magnitude of the shear stress greater at the moving plate \( (y = H) \) or at the stationary plate \( (y = 0) \)?

b. Derive an expression for the \( y \) position of zero shear stress.

c. Derive an expression for the plate speed \( u_L \) required to make the shear stress zero at \( y = 0 \).

2.39 Consider the ratio \( \mu_{100}/\mu_{50} \), where \( \mu \) is the viscosity of oxygen and the subscripts 100 and 50 are the temperatures of the oxygen in degrees Fahrenheit. Does this ratio have a value (a) less than 1, (b) equal to 1, or (c) greater than 1?
2.40 This problem involves a cylinder falling inside a pipe that is filled with oil, as depicted in the figure. The small space between the cylinder and the pipe is lubricated with an oil film that has viscosity $\mu$. Derive a formula for the steady rate of descent of a cylinder with weight $W$, diameter $d$, and length $l$ sliding inside a vertical smooth pipe that has inside diameter $D$. Assume that the cylinder is concentric with the pipe as it falls. Use the general formula to find the rate of descent of a cylinder 100 mm in diameter that slides inside a 100.5 mm pipe. The cylinder is 200 mm long and weighs 15 N. The lubricant is SAE 20W oil at 10°C.

2.41 The device shown consists of a disk that is rotated by a shaft. The disk is positioned very close to a solid boundary. Between the disk and the boundary is viscous oil.
   a. If the disk is rotated at a rate of 1 rad/s, what will be the ratio of the shear stress in the oil at 1 cm to the shear stress at 2 cm?
   b. If the rate of rotation is 2 rad/s, what is the speed of the oil in contact with the disk at 3 cm?
   c. If the oil viscosity is 0.01 N·s/m² and the spacing $y$ is 2 mm, what is the shear stress for the conditions noted in part (b)?

2.42 Some instruments having angular motion are damped by means of a disk connected to the shaft. The disk, in turn, is immersed in a container of oil, as shown. Derive a formula for the damping torque as a function of the disk diameter $D$, spacing $S$, rate of rotation $\omega$, and oil viscosity $\mu$.

2.43 One type of viscometer involves the use of a rotating cylinder inside a fixed cylinder. The gap between the cylinders must be very small to achieve a linear velocity distribution in the liquid. (Assume the maximum spacing for proper operation is 0.05 in.). Design a viscometer that will be used to measure the viscosity of motor oil from 50°F to 200°F.

Elasticity and Volume Changes

2.44 The bulk modulus of elasticity of ethyl alcohol is $1.06 \times 10^9$ Pa. For water, it is $2.15 \times 10^9$ Pa. Which of these liquids is easier to compress? Why?

2.45 A pressure of $2 \times 10^6$ N/m² is applied to a mass of water that initially filled a 2000 cm³ volume. Estimate its volume after the pressure is applied.

2.46 Calculate the pressure increase that must be applied to water to reduce its volume by 2%.

2.47 An open vat in a food processing plant contains 400 L of water at 20°C and atmospheric pressure. If the water is heated to 80°C, what will be the percentage change in its volume? If the vat has a diameter of 3 m, how much will the water level rise due to this temperature increase? Hint: In this case the volume change is due to change in temperature.

Surface Tension

2.48 Advanced texts define the surface tension $\sigma$ as an energy/area. Use primary dimensions to show that energy/area equals force/length.

2.49 Which of the following is the formula for the gage pressure within a very small spherical droplet of water:
   (a) $p = \sigma / d$, (b) $p = 4\sigma / d$, or (c) $p = 8\sigma / d$?

2.50 A spherical soap bubble has an inside radius $R$, a film thickness $t$, and a surface tension $\sigma$. Derive a formula for the pressure within the bubble relative to the outside atmospheric...
pressure. What is the pressure difference for a bubble with a 4 mm radius? Assume \( \sigma \) is the same as for pure water.

2.51 A water bug is suspended on the surface of a pond by surface tension (water does not wet the legs). The bug has six legs, and each leg is in contact with the water over a length of 5 mm. What is the maximum mass (in grams) of the bug if it is to avoid sinking?

2.52 A water column in a glass tube is used to measure the pressure in a pipe. The tube is 1/4 in. (6.35 mm) in diameter. How much of the water column is due to surface-tension effects? What would be the surface-tension effects if the tube were 1/8 in. (3.2 mm) or 1/32 in. (0.8 mm) in diameter?

2.53 Calculate the maximum capillary rise of water between two vertical glass plates spaced 1 mm apart.

2.54 What is the pressure within a 1 mm spherical droplet of water relative to the atmospheric pressure outside?

2.55 By measuring the capillary rise in a tube, one can calculate the surface tension. The surface tension of water varies linearly with temperature from 0.0756 N/m at 0°C to 0.0589 N/m at 100°C. Size a tube (specify diameter and length) that uses capillary rise of water to measure temperature in the range from 0°C to 100°C. Is this design for a thermometer a good idea?

2.56 Consider a soap bubble 2 mm in diameter and a droplet of water, also 2 mm in diameter, that are falling in air. If the value of the surface tension for the film of the soap bubble is assumed to be the same as that for water, which has the greater pressure inside it? (a) the bubble, (b) the droplet, (c) neither—the pressure is the same for both.

2.57 A drop of water at 20°C is forming under a solid surface. The configuration just before separating and falling as a drop is shown in the figure. Assume the forming drop has the volume of a hemisphere. What is the diameter of the hemisphere just before separating?

2.58 The surface tension of a liquid is being measured with a ring as shown in Fig. 2.6d. The ring has an outside diameter of 10 cm and an inside diameter of 9.5 cm. The mass of the ring is 10 g. The force required to pull the ring from the liquid is the weight corresponding to a mass of 16 g. What is the surface tension of the liquid (in N/m)?

Vapor Pressure

2.59 If a liquid reaches the vapor pressure, what happens in the liquid?

2.60 How does vapor pressure change with increasing temperature?

2.61 At a temperature of 60°F, what pressure must be imposed in order for water to boil.

2.62 Water is at 20°C, and the pressure is lowered until bubbles are noticed to be forming. What must the magnitude of the pressure be?

2.63 A student in the laboratory plans to exert a vacuum in the head space above a surface of water in a closed tank. She plans for the absolute pressure in the tank to be 10,400 Pa. The temperature in the lab is 20°C. Will water bubble into the vapor phase under these circumstances?

2.64 The vapor pressure of water at 100°C is 101 kN/m², because water boils under these conditions. The vapor pressure of water decreases approximately linearly with decreasing temperature at a rate of 3.1 kN/m²/°C. Calculate the boiling temperature of water at an altitude of 3000 m, where the atmospheric pressure is 69 kN/m² absolute.