The traffic light appeared early in the twentieth century. Incandescent bulbs were the primary source of illumination in traffic lights until the end of the twentieth century when traffic lights utilizing light-emitting diodes (LEDs) were developed. LEDs are semiconductor devices that emit light when an electrical current is passed through them. Where a single incandescent bulb was used for each color in the light, each color in an LED traffic light is realized with an array of LEDs. In comparison to traffic lights employing incandescent bulbs, LED traffic lights have the following advantages: (1) they last longer because they do not have a filament to burn out, (2) they are brighter, and (3) they are more energy efficient. As a result, many cities are replacing older traffic lights with the newer LED-based traffic lights.

Diodes are semiconductor devices employed in a variety of electrical circuits. They can be thought of as an electrical check.
valve—current can only flow through a diode in one direction. Diodes are utilized in a wide range of applications such as rectifiers, clamping circuits, and voltage regulators. Because of their widespread use, we will develop a basic understanding of the diode and learn how to analyze circuits containing diodes in this chapter.

17.1 Introduction

In this chapter we briefly introduce an electronic element that is widely used in circuits—the diode. Our discussion of the diode focuses on the construction of the device and four techniques that are used for modeling this element. The models are then used in circuit examples.

Materials can generally be classified as conductors, insulators, or semiconductors. This distinction is based on the resistivity of the material measured in Ω-m. For example, copper wire has a resistivity of $1.72 \times 10^{-8}$ Ω-m at room temperature. The resistivity of hard rubber is on the order of $10^{13}$ Ω-m, which places it in the category of an insulator. The resistivity of a semiconductor falls between that for conductors and insulators. A very common semiconductor is silicon. Intrinsic or pure silicon has a resistivity on the order of $10^3$ Ω-m. As a result, intrinsic silicon is a poor conductor of current.

A small and controlled addition of impurities to intrinsic silicon permits the resistivity to be changed over a wide range. This process of adding impurities to silicon is referred to as doping. Silicon is an element in column IV of the periodic table (shown partially in Fig. 17.1). If we dope silicon with elements from column III, such as boron, we obtain what is known as a p-type material. Doping with an element from column V, such as phosphorus, yields what is called an n-type material. P-type doping generates positive charge carriers, whereas n-type doping generates negative charge carriers. A carrier is a charged particle that can move about, creating current. A p-n junction is formed if the doping changes from n-type to p-type over a short distance. This p-n junction forms what is called a diode. The symbol and general structure of the diode are shown in Fig. 17.2. Diodes come in many sizes and shapes; some typical diodes are shown in the photograph in Fig. 17.3. The large diode in the upper-left corner of this photo is capable of conducting a current of 100 A.
Recall in Chapter 2 that we plotted the voltage versus current for a linear resistor and a light bulb. The plot of voltage versus current for a linear resistor was a straight line. We could just as easily have plotted current versus voltage for a linear resistor. It is more common to plot the current versus the voltage for a diode, and such a plot is shown in Fig. 17.4. Based on this plot, we can conclude that a diode is a nonlinear circuit element. There are three regions labeled on this plot: forward-bias, reverse-bias, and reverse-breakdown. In the forward-bias region, current flows through the diode from the anode to the cathode. Some positive voltage is required before any reasonable amount of current flow is obtained. This voltage will be referred to as the turn-on voltage \( V_{on} \); it is typically in the range of 0.6 — 0.7 V and is shown in Fig. 17.4. For large diodes like the one in the upper-left corner of Fig. 17.3, \( V_{on} \) can easily exceed 1 V. The plot in Fig. 17.4 indicates that the current flowing through the diode is zero in the reverse-bias region. In actuality the current is not zero, but a small negative value referred to as the reverse saturation current \( I_s \) as shown in Fig. 17.5. The voltage across the diode in the reverse-bias region is negative. If the magnitude of the voltage across
the diode in the reverse-bias region exceeds the voltage \( V_Z \), the diode enters the reverse-breakdown region where the current flowing through the diode increases rapidly unless it is limited by external circuitry. The power dissipated in the diode in this region can be quite high, resulting in failure of the diode. The voltage \( V_Z \) is referred to as the breakdown voltage of the diode. This is not a desirable operating region for most diodes; however, some diodes such as Zener diodes (to be discussed later) are actually designed to operate in this region.

We can think of a diode as a kind of electric valve. In the forward-bias region, the diode conducts current from the anode to cathode, and a small voltage is dropped across the diode. In the reverse-bias region, the diode blocks the flow of current from cathode to anode unless the breakdown voltage is exceeded.

The current–voltage characteristic for the forward-bias and reverse-bias regions in Fig. 17.4 can be described by a mathematical equation called the diode equation

\[
i_D = I_s \left( e^{qV_D/kT} - 1 \right)
\]

where \( i_D \) and \( V_D \) are the diode current and voltage as defined in Fig. 17.2. \( I_s \) is the reverse saturation current in Fig. 17.5, \( q \) is the charge of an electron \((1.60 \times 10^{-19} \text{ C})\), \( k \) is Boltzmann’s constant \((1.38 \times 10^{-23} \text{ J/K})\), \( T \) is temperature in Kelvin, and \( n \) is the ideality factor. Typical values for \( I_s \) and \( n \) are around \( 10^{-14} \) A and 1, respectively. However, in diodes designed for large currents such as the one in Fig. 17.3, approximately \( 10^{-10} \) A and 2 are more representative. At room temperature, \( q/kT \) is approximately 39. Therefore, the diode equation can be expressed as

\[
i_D = I_s \left( e^{qV_D/kT} - 1 \right)
\]

In the forward-bias condition, the exponential term has a positive exponent, and this term dominates the function. Thus, the \( i \)-\( v \) characteristic is exponential in nature. However, some positive voltage is required before any reasonable amount of current flow is obtained. In the reverse-bias region, the negative value of \( V_D \) makes the exponential term negative, so \( i_D \) asymptotically approaches the value of \(-I_s\). The diode equation in Eq. (17.1) forms the core of the diode model used in simulators such as PSPICE.

### 17.2 Modeling Techniques

All of the circuits that we have analyzed to this point have been linear circuits. Diodes are nonlinear circuit elements, so we must be careful when analyzing circuits containing these devices. In previous chapters, we introduced models for circuit elements such as resistors, capacitors, inductors, voltage sources, and current sources. Diodes can be modeled using two distinct approaches: a nonlinear model based on Eq. (17.1) or a linear approximation to the current–voltage characteristics of the device. There are three popular linear models for
the diode: the ideal model, the constant-voltage model, and the piecewise-linear model. In the ideal model, with diode symbol and \(i-v\) curve illustrated in Figs. 17.6a and 17.6b, the device is treated as a simple switch. When acting like a closed switch, as shown in Fig. 17.6c, the diode is said to be forward-biased with \(v_D\) zero and \(i_D\) positive. Alternatively, when reverse-biased, the condition in Fig. 17.6d, \(i_D\) is zero, \(v_D\) is negative, and the “switch” is open. Thus, the ideal diode passes current from anode to cathode and blocks current in the opposite direction.

![Figure 17.6](image)

(a) Diode symbol, (b) \(i-v\) characteristics for an ideal diode, and (c) equivalent circuits in forward bias and (d) in reverse bias.

The constant-voltage model, a modification of the ideal model, is a very popular model for pen-and-paper analysis. Here, the diode is modeled as a dc voltage source in series with an ideal diode, as shown in Fig. 17.7a. The resulting \(i-v\) characteristic is shown in Fig. 17.7b. When forward-biased, \(v_D = V_{on}\) and is typically chosen at 0.6 or 0.7 V. Note that \(v_D\) values below \(V_{on}\) produce no current. Therefore, the diode is reverse-biased for \(v_D < V_{on}\).

![Figure 17.7](image)

(a) The constant-voltage model equivalent circuit and (b) the corresponding \(i-v\) characteristic.

Neither the ideal nor the constant-voltage model treats the finite slope of the \(i-v\) curve in forward-bias. The piecewise-linear model does so by adding a series resistance to the constant voltage model, as shown in Fig. 17.8a. Figure 17.8b shows the resulting \(i-v\) characteristic.

![Figure 17.8](image)

(a) The piecewise-linear model equivalent circuit and (b) the corresponding \(i-v\) characteristic.
where the slope is \(1/R_d\). The diode is reverse-biased with no current until \(v_D\) reaches \(V_F\). At that point, the diode is forward-biased and current increases linearly with \(v_D\).

A comparison of the \(i-v\) characteristics of these four models is shown in Fig. 17.9. We see that the accuracy of the models with respect to the diode equation improves as we progress from the ideal to the constant voltage and finally to the piecewise-linear model. Unfortunately, this accuracy is bought at the price of model complexity. Tradeoffs such as these are common in engineering and should not be viewed pessimistically. Instead, consider the options a blessing, investigate the tradeoffs, and identify the strengths and weaknesses of each option. Then use your knowledge to choose the best model for the analysis at hand.

**EXAMPLE 17.1**

* Using the ideal diode model, find \(V_1\) in the circuits in Fig. 17.10.

**SOLUTION**

In Fig. 17.11a, the voltage source produces a clockwise current, which forward-biases the diode. Therefore, by inspection, \(V_1 = 12\) V. In Fig. 17.11b, the current source should produce a clockwise current, and \(I\) should be positive. However, the diode is reverse-biased, conducting no current, and all the source current flows through the resistor. Thus, the voltage is \(V_1 = (0.1)(100) = 10\) V.
In Fig. 17.11c, the voltage source produces clockwise currents, which will forward-bias the diode. Under this condition, the 3-kΩ resistor is shorted by the diode and, hence, \( V_1 = 4 \) V.

In Fig. 17.11d, the voltage source will produce counterclockwise currents. Since the diode cannot conduct in that direction, it is reverse-biased, and \( V_1 \) can be found by voltage division as

\[
V_1 = -4 \left[ \frac{1000}{1000 + 3000} \right] = -1 \text{ V}
\]

\[\text{Figure 17.11}\]
Equivalent circuits for the circuits in Fig. 17.10 using the ideal-diode model.

Diodes are commonly used to protect sensitive circuitry from excessive over/under-voltage conditions. The circuit in Fig. 17.12 is typical where \( v_S(t) \) and \( R_S \) make up the Thévenin equivalent of the driving network. Using the ideal-diode, constant-voltage, and piecewise-linear models for the diode, let us determine the maximum and minimum voltages at the input to the protected circuit.

\[\text{Figure 17.12}\]
A diode circuit that provides voltage protection.
For simplicity, we will assume that the protected circuit has infinite input resistance; that is, if the diodes are reverse-biased, then \( v_s(t) = v_{in}(t) \). Diodes \( D_1 \) and \( D_2 \) provide over/under-voltage protection as follows. Suppose \( v_s(t) \) drops below ground to the point that \( D_2 \) becomes forward-biased. Figure 17.13 shows the situation for the constant-voltage model. For the under-voltage condition, \( v_{in}(t) \) is “pinned” at \(-V_{on}\), and the circuit is protected. Should \( v_s(t) \) exceed \( V_{dc} \), then \( D_1 \) will be forward-biased and \( v_{in}(t) \) will be pinned at \( V_{dc} + V_{on}\).

If we use the ideal-diode model, \( v_{in}(t) \) pins exactly at ground and \( V_{dc} \). For the piecewise-linear model, the circuit in Fig. 17.14 models the under-voltage scenario. By superposition, \( v_{inu} \) can be written as

\[
v_{inu} = \frac{v_s R_d}{R_d + R_s} - \frac{V_{F} R_s}{R_d + R_s}
\]

A similar analysis in the over-voltage case yields

\[
v_{ino} = \frac{v_s R_d}{R_d + R_s} + \frac{(V_{dc} + V_{F}) R_s}{R_d + R_s}
\]

where \( v_{inu} \) and \( v_{ino} \) denote the under- and over-voltage conditions, respectively. Plots for \( v_{in} \) versus \( v_s \) for \( V_{dc} = 3 \) V are shown in Fig. 17.15 for the ideal, constant-voltage (\( V_{on} = 0.7 \) V), and piecewise-linear model (\( V_F = 0.6 \) V, \( R_d = 10 \) Ω, and \( R_s = 40 \) Ω).

**Figure 17.13**

Using the constant-voltage model: (a) under-voltage and (b) over-voltage.

**Figure 17.14**

Using the piecewise-linear model: (a) under-voltage and (b) over-voltage.
In the previous section, we analyzed circuits containing diodes using the ideal-diode, constant-voltage, and piecewise-linear models. Now let’s consider analyzing the simple circuit in Fig. 17.16 using the diode equation. Since any complex linear circuit can be reduced to its Thévenin equivalent, as was illustrated in Chapter 5, the network in Fig. 17.16 is a general representation of any linear circuit containing only a single diode.

Applying Kirchhoff’s voltage law to this circuit yields

\[ V_{oc} = R_{Th} i_D + v_D \]  \hspace{1cm} (17.3)

Because \( V_{oc} \) and \( R_{Th} \) are known quantities, we have one equation in two unknowns: \( i_D \) and \( v_D \). These unknowns are also related by the diode equation in Eq. (17.1). We must simultaneously solve Eq. (17.1) and Eq. (17.3) to determine the voltage and current in Fig. 17.16.

The simultaneous solution of Eqs. (17.1) and (17.3) can be obtained graphically or numerically. If the \( i-v \) curve for the diode is only available as a plot, then we must use a graphical approach. The two equations are plotted, and their intersection gives us the voltage and current for the circuit. This approach is also referred to as a load-line analysis, where Eq. (17.3) is the load line. Recall from Chapter 3 that we used nodal and loop analysis to write a set of linear equations for a circuit. The simultaneous solution of this set of linear equations can be obtained using Gaussian elimination, matrix inversion, or MATLAB. Because the diode equation is a nonlinear equation, we are now tasked with the simultaneous solution of one linear equation and one nonlinear equation. As a result, the techniques we utilized earlier
for solving a set of linear equations may not be employed here. We must rely on numerical techniques to find the solution of the circuit in Fig. 17.16 when the diode equation is used.

Let’s first describe how to solve the circuit of Fig. 17.16 using the graphical approach. Figure 17.17 is a plot of the diode $i$-$v$ curve and the load line. The load line is the equation for a straight line, and two points are needed to graph a straight line. One convenient point can be obtained by setting $i_D = 0$, which yields $v_D = V_{oc}$. A second convenient point is obtained by setting $v_D = 0$, which yields $I_{sc} = V_{oc}/R_{Th}$. Then a straight line is drawn connecting these points. The intersection of the load line and the diode $i$-$v$ curve is the solution for the current and voltage, which is represented by $i_{D1}$ and $v_{D1}$.

**EXAMPLE 17.3** Using the diode $i$-$v$ curve in Fig. 17.18b, utilize the graphical approach to solve for the voltage and current in the circuit of Fig. 17.18a.
If we have only a plot of the $i$-$v$ curve for a diode, then our only solution method is the graphical approach. A mathematical solution is possible when we have parameters for the diode equation in Eq. (17.1) or Eq. (17.2). Again, our task is to simultaneously solve the load line and diode equations. Substituting Eq. (17.2) in Eq. (17.3) yields

$$V_{oc} = R_{Th} I_s \left[ e^{\frac{v_D}{R_{Th}}} - 1 \right] + v_D$$  \hspace{1cm} 17.4

Note that this is not a linear equation! This equation is referred to as a transcendental equation. It does not have a closed-form solution, and it must be solved numerically. Many numerical techniques exist to find a solution to Eq. (17.4); they have been implemented in modern scientific calculators and software such as MATLAB. As such, we will not present techniques to solve Eq. (17.4) but merely use existing computational tools to find our solution.

Equation (17.4) was developed by substituting the expression for the diode current into the load-line equation. An alternative approach is to solve the diode equation for the diode

---

**Figure 17.19**

Graphical solution for Example 17.3.

Using Kirchhoff’s voltage law, the load line is

$$5 = 1000i_D + v_D$$

The open-circuit voltage is 5 V, and the short-circuit current is $5/1000 = 5$ mA. One endpoint for our load line is the point $(0, 0.005 \, \text{A})$. The $x$-axis on the plot in Fig. 17.18b does not extend beyond 1 V. As a result, we are not able to use the point $(V_{oc} = 5 \, \text{V}, 0)$ to draw the load line. We need to utilize another point on our load line. Let’s take the load line and solve for $i_D$:

$$i_D = \frac{5 - v_D}{1000}$$

Using $v_D = 1 \, \text{V}$ yields $i_D = 4$ mA. Our two points for plotting the load line are $(0, 0.005 \, \text{A})$ and $(1 \, \text{V}, 0.004 \, \text{A})$. In Fig. 17.19, the load line is plotted with the diode $i$-$v$ curve. The intersection of the two curves represents the solution. The current $i_D$ is approximately 4.3 mA, and the voltage $v_D$ is approximately 0.69 V.
voltage \( v_D \) and substitute for it in the load-line equation. Let’s solve Eq. (17.2) for the diode voltage. Our first step is to divide both sides of Eq. (17.2) by \( I_S \), which yields
\[
\frac{i_D}{I_S} = e^{\frac{v_D}{n}} - 1
\]
Now add 1 to both sides of the equation:
\[
\frac{i_D}{I_S} + 1 = e^{\frac{v_D}{n}} \tag{17.6}
\]
Let’s take the natural logarithm of both sides of this equation:
\[
ln \left( \frac{i_D}{I_S} + 1 \right) = \frac{v_D}{n} \tag{17.7}
\]
Rearranging this equation yields an expression for \( v_D \) in terms of \( i_D \):
\[
v_D = \frac{1}{n} \ln \left( \frac{i_D}{I_S} + 1 \right) \tag{17.8}
\]
Substituting this equation into the load line gives us an alternative expression to Eq. (17.4).
\[
V_\infty = R_T i_D + \frac{1}{n} \ln \left( \frac{i_D}{I_S} + 1 \right) \tag{17.9}
\]
This is also a transcendental equation that must be solved numerically. Either Eq. (17.4) or Eq. (17.9) could be employed to find the solution to the circuit of Fig. 17.16.

**EXAMPLE 17.4**

Solve for the voltage and current in the circuit of Fig. 17.18 using the diode equation in Eq. (17.2). Assume that \( I_S = 10^{-14} \) A.

**SOLUTION**

Referring to Fig. 17.18 and plugging circuit values into Eq. (17.4) yields
\[
5 = 1000\left(10^{-14}\right)\left(1 - e^{10^{\frac{v_D}{n}} - 1}\right) + v_D
\]
\[
5 = 10^{-11}e^{10^{\frac{v_D}{n}} - 1} + v_D
\]
Let’s rearrange into the following form:
\[
f(v_D) = 5 - 10^{-11}e^{10^{\frac{v_D}{n}} - 1} - v_D = 0
\]
Our task is to find the value of \( v_D \), which makes this function equal 0. We begin by guessing a value for \( v_D \) and evaluating the function. If it evaluates to 0, then we guessed the correct answer and do not need to perform any additional calculations. If our evaluation is not 0, then we need to come up with a new guess for \( v_D \) and evaluate the function again. Numerical techniques for solving transcendental equations utilize this same procedure. You provide an initial guess to the solution. A function is evaluated to determine if the current guess satisfies that function. If it does not, the numerical techniques provide a systematic way to determine the next guess. That guess is then checked to see if it is the solution to the function.

Suppose we wanted to use Excel to determine a solution to our function. We know that the diode voltage is going to be around 0.6 V. Let’s evaluate the function for values of \( v_D \) between 0.6 and 0.7 V. Figure 17.20a illustrates this procedure. The first column contains values for the diode voltage, and the second column contains values of the function \( f(v_D) \). Note that our solution must lie between 0.68 V and 0.69 V as the sign of \( f(v_D) \) changes from positive to negative within that range. Figure 17.20b contains the calculation results for diode voltages between 0.68 V and 0.688 V. This data reveals that our answer is between 0.686 V and 0.687 V. We could continue our search by examining diode voltages between these two values, which would require us to add more digits to our guesses. For example, \( v_D = 0.68692 \) V yields \( f(v_D) = 0.0009 \). Even though we increased the precision of our guess, we have not yet reached zero. This is a characteristic of the solution of transcendental equations. Typically, the solution process is stopped when \( |f(v_D)| \) evaluated at a particular guess is less than some small value, which is often referred to as a tolerance.
For example, we might decide for the present problem to stop when \( |f(v_D)| \) is less than 0.02. Using this condition and examining the data in Fig. 17.20b yields 0.687 V as the diode voltage. We can now calculate the diode current using Eq. (17.2):

\[
  i_D = 10^{-14} [e^{39(0.687)} - 1] = 4.33 \text{ mA}
\]

<table>
<thead>
<tr>
<th>( v_D )</th>
<th>( f(v_D) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.61</td>
<td>4.0629</td>
</tr>
<tr>
<td>0.62</td>
<td>3.9016</td>
</tr>
<tr>
<td>0.63</td>
<td>3.6682</td>
</tr>
<tr>
<td>0.64</td>
<td>3.3282</td>
</tr>
<tr>
<td>0.65</td>
<td>2.8308</td>
</tr>
<tr>
<td>0.66</td>
<td>2.1010</td>
</tr>
<tr>
<td>0.67</td>
<td>1.0278</td>
</tr>
<tr>
<td>0.68</td>
<td>-0.5526</td>
</tr>
<tr>
<td>0.69</td>
<td>-2.8819</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( v_D )</th>
<th>( f(v_D) )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>4.2546</td>
</tr>
<tr>
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</tr>
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</tr>
<tr>
<td>0.70</td>
<td>-2.8819</td>
</tr>
</tbody>
</table>

Let’s also employ MATLAB to solve for the diode voltage and current in our example.

We can utilize an inline function and then the \texttt{fzero} command to find a solution. The first line below uses the inline command to define the inline function \( f_vd \). The \texttt{fzero} command takes our function \( f_vd \) and an initial guess (0.6) and determines that a diode voltage of 0.6869 V makes the function \( f_vd = 0 \). The diode equation is then used to calculate the corresponding diode current of 4.3 mA. The solution proceeds as follows:

\[
>> f_vd = \text{inline} \left( '5-1e-11*(exp(39*vd)-1)-vd', 'vd' \right)
\]

\[
f_vd = \text{Inline function}
\]

\[
f_vd(vd)=5-1e-11*(exp(39*vd)-1)-vd
\]

\[
>> x = \text{fzero}(f_vd, 0.6)
\]

\[
x =
0.6869
\]

\[
>> id = 1e-14*(exp(39*0.6869)-1)
\]

\[
id =
0.0043
\]

Another approach is to employ the anonymous function capability of MATLAB. An anonymous function is created using the first line below. The @ symbol indicates to MATLAB that \( f_vd \) is a function. Immediately after the @ symbol is the input to the function. Then, the function itself is defined. The \texttt{fzero} command is again utilized to find our solution. The only difference in these two cases is in the definition of the function \( f_vd \). The syntax for the inline function is slightly more complicated than that for the anonymous function. In this case, the solution is derived as follows:

\[
>> f_vd = @(vd)5-1e-11*(exp(39*vd)-1)-vd
\]

\[
f_vd = \quad @(vd)5-1e-11*(exp(39*vd)-1)-vd
\]

\[
>> x = \text{fzero}(f_vd, 0.6)
\]

\[
x =
0.6869
\]

\[
>> id = 1e-14*(exp(39*0.6869)-1)
\]

\[
id =
0.0043
\]
Diode Rectifiers

One common application for diodes is rectification, or the conversion of an ac voltage to a dc voltage. The circuit in Fig. 17.21 contains an ac voltage source, a diode, and a resistor. Let’s employ the ideal-diode model to examine operation of this circuit.

Referring back to Fig. 17.6, we see that \( i_D > 0 \) and \( v_D = 0 \) for forward-bias and \( i_D = 0 \) and \( v_D < 0 \) for reverse-bias. The current in Fig. 17.21 can only be positive when \( v(t) > 0 \) as shown in Fig. 17.22a. A negative value of \( v(t) \) would cause current to flow counterclockwise in the circuit of Fig. 17.21. However, our ideal diode dictates that current can only flow in the clockwise direction of Fig. 17.21. As a result, the current must be zero as shown in Fig. 17.22b. The operation of the circuit in Fig. 17.21, referred to as a half-wave rectifier, is illustrated by the plots in Fig. 17.23. Note that the output voltage, \( v_0(t) \), can only be positive or zero in this circuit. Only the positive half of the sinusoidal input voltage appears across the load resistor; the negative half of the input voltage is clipped off. This circuit converts an ac, or bidirectional, voltage to a dc, or unidirectional, voltage. The Fourier series for the output voltage waveform is given in Table 15.2.

The circuits in Fig. 17.24 are referred to as full-wave rectifiers. In these circuits, two diodes conduct simultaneously. When \( v(t) > 0 \), diodes D1 and D4 conduct. Diodes D2 and D3 conduct when \( v(t) < 0 \). The operation of these circuits, referred to as full-wave rectifiers, is illustrated by the plots in Fig. 17.25. Comparing Figs. 17.23 and 17.25, we can see that the full-wave rectifier does not clip off the negative half of the input voltage. Instead, it is inverted and appears as part of the output voltage.

The Fourier series for the full-wave rectifier’s output voltage is also given in Table 15.2. Let’s compare the average values for the half-wave and full-wave rectifiers. Recall that the average value is \( a_0 \) and is calculated using Eq. (15.15). For the half-wave rectifier, the average value is \( \frac{A}{\pi} \), where \( A \) is the peak value of the sinusoidal input voltage. For the full-wave rectifier, \( a_0 = \frac{2A}{\pi} \), which is twice the value for the half-wave rectifier. The period of the output voltage waveform in Fig. 17.25 is one-half of the period of output voltage in Fig. 17.23. The definition of the rms value for a function was given in Chapter 9. In Example 9.7 of that chapter, we determined that the rms value of a sinusoidal waveform, such as the input voltage in Fig. 17.25, is 0.707\( A \). The rms value of the output voltage in Fig. 17.25 is...
**Figure 17.23**
Waveforms for a half-wave rectifier: (top) input voltage $v(t)$, (middle) output voltage $v_o(t)$, and (bottom) diode voltage $v_D(t)$.

**Figure 17.24**
Full-wave rectifier circuits.
also 0.707 A. Thinking about the definition of rms, can you verify that the rms value is 0.707 A without performing the actual calculation?

Examining the output voltage waveform of Fig. 17.23, we can see that the output voltage varies between 0 and the peak value. Some loads connected to the output of the rectifier may not be able to tolerate this variation in voltage and require a more constant voltage. Referring back to the Fourier series for this voltage waveform in Table 15.2, we note that this series consists of an average value or dc term and a number of sinusoidal terms. If our goal is to have a near constant output voltage, then we can utilize a low-pass filter to attenuate the amplitude of the sinusoidal terms. This is easily accomplished by adding a capacitor across the load resistor as shown in Fig. 17.26. Waveforms for this circuit were created using PSPICE and are plotted in Fig. 17.27. The diode begins to conduct when $V_m \sin \omega t = v_0$ and continues to conduct until the peak value $V_m$ is reached. The equivalent circuit of Fig. 17.28a can be utilized to analyze the half-wave rectifier when the diode is conducting. When $V_m \sin \omega t < v_0$, the diode is blocking, and the capacitor supplies the load $R$ as shown in Fig. 17.28b.

The voltage $V_r$ in Fig. 17.27 is referred to as the ripple voltage and depends on the values of $R$ and $C$. Referring back to Chapter 7, we can describe the output voltage in Fig. 17.28b by

$$v_0(t) = V_m e^{-t'/R C} = V_m e^{-t'/\tau}$$

where $t' = 0$ corresponds to turn off of the diode. We can see from Fig. 17.27 that the diode is off most of the time and that the capacitor holds up the output voltage. Let’s define $t'_\text{on}$ as the value of $t'$ at which the diode begins to conduct. Now $t'_\text{on}$ is less than the period, $T = 2\pi/\omega$, of the ac source. As an approximation, let’s assume that $t'_\text{on} = T$ in Eq. (17.10). The ripple voltage will be small as long as $T \ll \tau = R C$. For a given value of $R$, the ripple voltage can be reduced by increasing the value of $C$. In addition, increasing $C$ also increases $t'_\text{on}$. 

Figure 17.25

Waveforms for a full-wave rectifier: (top) input voltage $v(t)$, (middle) output voltage $v_0(t)$, and (bottom) diode voltage $v_D(t)$. 

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Also, 0.707 A. Thinking about the definition of rms, can you verify that the rms value is 0.707 A without performing the actual calculation?
In Fig. 17.26, let's assume that $v(t) = V_m \sin(\omega t)$ volts and $R = 100 \ \Omega$ and calculate the value of $C$ required to hold the output voltage to 168 V. The period for our ac source is 16.67 ms. Using our approximation, Eq. (17.10) becomes

$$C = \frac{168}{170e^{-\frac{100}{100}} \cdot 0.0166}$$

Solving this equation for $C$ yields $C = 14.1 \ \text{mF}$. Let's make a couple of observations at this point. First, this is a very large value of capacitance. Typically, an electrolytic capacitor is utilized for $C$ because of the large capacitance value and required voltage rating in excess.
of 170 V. Second, since we employed the approximation that $r_{on} = T$, the calculated capacitance value is slightly larger than the value required in the actual circuit.

### 17.5 Zener Diodes

Referring back to Fig. 17.4, we note that there are three regions on this plot: forward-bias, reverse-bias, and reverse-breakdown. Our discussion and analyses so far have focused on the forward-bias and reverse-bias regions. Let’s consider operation in the reverse-breakdown region where the diode voltage is almost constant and approximately independent of the current flowing through the diode. A Zener diode is a particular type of diode that is designed to operate in the reverse-breakdown region. Figure 17.29 is a simple circuit that contains a Zener diode. Note that a different symbol is used for a Zener diode. Also, the orientation of the voltage and current is the same as that in Fig. 17.2. Again, we are representing the circuit connected to the Zener diode with a Thévenin equivalent circuit.

Applying Kirchhoff’s voltage law to the circuit in Fig. 17.29 yields

$$-V_{oc} = R_{Th}i_D + v_D \quad 17.11$$

This is our load-line equation with $-V_{oc}$. The solution to this circuit can be found using a graphical approach as shown in Fig. 17.30. We are assuming that $V_{oc} > V_Z$. This must be true for the diode to be operating in the reverse-breakdown region.

Earlier in this chapter, we presented three circuit models for a diode operating in the forward-bias and reverse-bias regions. Two of these models, constant-voltage and piecewise-linear, can be adapted to model the Zener diode as shown in Fig. 17.31. In comparison to the models in Figs. 17.7a and 17.8a, the voltage source in these models is oriented differently with respect to $v_D$ and $i_D$ because of operation in the reverse-breakdown region. In the piecewise-linear model, $R_Z$ is typically less than 100 $\Omega$.

Let’s replace the Zener diode in the circuit of Fig. 17.29 with the constant-voltage model as shown in Fig. 17.32. The current $I_Z$ can be calculated using

$$I_Z = \frac{V_{oc} - V_Z}{R_{Th}} \quad 17.12$$

The power absorbed by the Zener diode is

$$p_Z = V_Z I_Z = \frac{V_Z(V_{oc} - V_Z)}{R_{Th}} \quad 17.13$$
For Zener diode operation, we require $V_{oc} > V_Z$. A resistance must be connected in the circuit between the two voltage sources of unequal value to limit the current flow between them. When using Zener diodes, we will connect a resistor in the circuit to limit the current flow into the diode. Examining Eq. (17.13) reveals that the amount of power absorbed or dissipated by the Zener diode is directly proportional to the voltage difference and inversely proportional to the resistance. What value of resistance is needed in the circuit? Like resistors, Zener diodes have power ratings that specify the maximum amount of power that can be dissipated in them. The resistance will need to be large enough to limit the power dissipation in the Zener diode below its rating.

\[ V_{oc} > V_Z \]

\[ R = \frac{V_{oc} - V_Z}{I_Z} \]

\[ P = \frac{(V_{oc} - V_Z)^2}{R} \]

**Solution**

Solve for the current $I_Z$ and the power dissipated in the Zener diode in the circuit of Fig. 17.33 using the constant-voltage and the piecewise-linear models for the Zener diode. Assume $V_Z = 5\, \text{V}$ and $R_Z = 100\, \Omega$.

**Example 17.5**

In Fig. 17.34a, the constant-voltage model has been substituted for the Zener diode. The current flowing in this circuit is

\[ I_Z = \frac{15 - 5}{5k} = 2\, \text{mA} \]

The power dissipated in the Zener diode is

\[ P_Z = (5)(2 \times 10^{-3}) = 10\, \text{mW} \]

Using the piecewise-linear model in Fig. 17.34b, we find that the current is

\[ I_Z = \frac{15 - 5}{5k + 0.1k} = 1.96\, \text{mA} \]
Zener diodes are often utilized in voltage regulator circuits like that shown in Fig. 17.35. If \( I_z > 0 \), the voltage across the load, modeled by resistor \( R_L \), is approximately \( V_Z \). Of course, \( V_S > V_Z \). This circuit could be utilized to supply a 5-V load from a 15-V source. Using Thévenin’s Theorem, we can reduce the circuit in Fig. 17.35 to that in Fig. 17.29 or Fig. 17.33. Our previous analyses in this section can then be applied to examine the operation of the voltage regulator circuit. The circuits for calculating the open-circuit voltage and Thévenin resistance are shown in Fig. 17.36. The open-circuit voltage is given by

\[
V_{oc} = \frac{R_L}{R_s + R_L} V_S \tag{17.14}
\]

and the Thévenin resistance is calculated as

\[
R_{Th} = \frac{R_L R_s}{R_L + R_s} \tag{17.15}
\]

Earlier in this section, we determined that \( V_{oc} > V_Z \) for the Zener diode to be operating in the reverse-breakdown region. Substituting for \( V_{oc} \) using Eq. (17.14) yields

\[
\frac{R_L}{R_s + R_L} V_S > V_Z \tag{17.16}
\]

The power dissipated in the Zener diode using this model is

\[
p_Z = (5)(1.96 \times 10^{-3}) + (1.96 \times 10^{-3})(100) = 0.18 \text{ mW}.
\]

The voltage \( V_1 \) across the Zener diode, the voltage across both \( R_S \) and \( V_Z \), is

\[
V_1 = (1.96 \times 10^{-3})(100) + 5 = 5.196 \text{ V}.
\]
This equation can be rearranged to determine a minimum value for \( R_L \) given \( V_S, V_Z, \) and \( R_S \):

\[
R_L > \frac{R_S}{V_S - 1} = R_{L\text{min}} \quad 17.17
\]

Alternately, we could rearrange this equation to solve for a maximum value of \( R_S \) given \( V_S, V_Z, \) and \( R_L \):

\[
R_S < R_L \left( \frac{V_S}{V_Z} - 1 \right) = R_{S\text{max}} \quad 17.18
\]

The voltage regulator in Fig. 17.37 is utilized to supply a 5-V load from a 15-V supply. Using a piecewise-linear model with \( V_Z = 5 \text{ V} \) and \( R_Z = 100 \text{ } \Omega \) for the Zener diode, calculate \( V_L, I_L, \) and \( I_Z \).

In Fig. 17.38a, the piecewise-linear model for the Zener diode has been substituted into the circuit of Fig. 17.37. Using Eqs. (17.14) and (17.15), we can calculate \( V_{oc} \) and \( R_{Th} \) as follows:

\[
V_{oc} = \frac{2k}{3k + 2k} 15 = 6 \text{ V} \quad R_{Th} = \frac{(2k)(3k)}{2k + 3k} = 1200 \Omega
\]

Following the analysis presented in Example 17.5, the Thévenin equivalent circuit is shown in Fig. 17.38b. The current flowing in this circuit is

\[
I_L = \frac{6 - 5}{1300} = 0.77 \text{ mA}
\]

and \( V_L = (0.77 \times 10^{-3})(100) + 5 = 5.077 \text{ V} \). Using the circuit in Fig. 17.38a, \( I_L = 5.077/2k \) = 2.54 mA.

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**EXAMPLE 17.6**

Voltage regulator for Example 17.6

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**SOLUTION**

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**Figure 17.37**

Voltage regulator for Example 17.6

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**Figure 17.38**

(a) Voltage regulator with piecewise-linear Zener model and (b) Thévenin equivalent circuit for voltage regulator.
REFERENCES