12.9 Natural Convection in Vertical Channels

In some applications, natural convection occurs from two closely-spaced vertical plates open at the top and bottom, as shown in Figure S12-1. An array of rectangular fins used in heat sinks may be idealized by this geometry. Another common application is the stack of printed circuit boards inside a computer. The plates are hotter than the surrounding fluid, and buoyancy forces induce a vertical flow. Heated fluid rises and exits at the top of the channel, while cool fluid is drawn into the bottom. This natural draft is termed the “chimney effect.”

Boundary layers form on each wall of the channel and increase in thickness with vertical height. If the channel is long enough, the boundary layers eventually join at the center as in internal forced convection and the flow at the exit is fully developed. If the channel is short, the boundary layers are still developing at the exit. The development of the boundary layers is influenced by the ratio of plate spacing, $S$, to plate height, $L$. If the plates are far apart, so that $S/L$ is very large, they behave like isolated vertical plates in an infinite fluid. If they are close together, so that $S/L$ is small, the boundary layers join and the flow is fully developed at the exit. Because of its important effect on the character of the flow, the ratio $S/L$ appears in the correlations that follow.

Natural convection in a channel depends on the boundary conditions at the walls. Four different cases have been investigated: both walls at a constant temperature, both walls with constant applied heat flux, one wall insulated and one wall at constant temperature, and one wall insulated and one wall with constant applied heat flux. Correlations are given for these four cases in Table S12-1. If temperature is specified on either wall, the correlation is written in terms of the Rayleigh number defined as

![Figure S12-1 Natural convection in a vertical parallel plate channel](attachment:image.png)
\[ Ra_s = Gr_s Pr = \frac{g \beta \rho \gamma (T_u - T_f)}{\mu} S^3 \]

The Nusselt number and heat transfer rate are

\[ Nu_s = \frac{hS}{k} \quad \dot{Q} = hA(T_u - T_f) \]

Properties are evaluated at the film temperature, \( i.e. \), the average of the wall and fluid temperatures.

If heat flux is specified on one or both walls, there is no obvious wall temperature to use in the Rayleigh and heat rate equations. The correlations given in Table S12-1 are based on the maximum wall temperature, \( T_e \), which occurs at the exit of the channel and the modified Rayleigh number defined as

\[ Ra_s' = \frac{g \beta q' \rho^2 c_p S^4}{\mu k^2} \]

For the constant heat flux case, the Nusselt number and heat transfer rate are

\[ Nu_s = \frac{hS}{k} \quad \dot{Q} = hA(T_u - T_f) \]

Fluid properties are evaluated at the average of \( T_e \) and \( T_f \).

The spacing of the plates determines the size of the heat transfer coefficient. If plates are so far apart that the boundary layers do not interfere, heat transfer is a maximum. As the plates are moved closer together, the boundary layers interact and the rate of heat transfer decreases. In closely-spaced plates, viscous forces result in lower flow velocity which limits heat transfer.

In some design applications, the goal is to remove as much heat as possible in a given volume. For example, if a finned array is used as a heat sink to cool a power transistor, the heat sink should be as small as possible. If the plates of the finned array are far apart, heat transfer coefficient is high, but the total surface area of the plates per unit volume is low. Conversely, if the plates are close together, heat transfer coefficient is low, but the total surface area per unit volume is high. Heat transfer is proportional to the product of heat transfer coefficient and surface area, \( \dot{Q} = hA(T_u - T_f) \); therefore, there exists an optimal spacing at which \( \dot{Q} \) is maximum. This spacing is given as \( S_{opt} \) in Table S12-1.

If temperature is specified on one or both plates, \( S_{opt} \) gives the maximum heat transfer for a given temperature difference between plate and fluid. In the case of constant heat flux, spacing the plates closer simply increases the total heat per unit volume. However, as the plates are brought closer together, the plate temperature increases. So in the case of a specified heat flux boundary condition, \( S_{opt} \) gives the maximum heat transfer per unit temperature difference between plate and fluid.

In some cases, the goal is simply to transfer the maximum possible heat from each plate. This occurs when plates are far enough apart so that boundary layers do not interfere. In Table S12-1, the spacing \( S_{max} \) is the minimum distance between plates with independent boundary layers. For any spacing greater than or equal to \( S_{max} \), the plates behave like isolated vertical plates.
Table S12-1 Correlations for natural convection in vertical parallel plate channels

\[
Nu_s = \left\{ \frac{576}{\left[ Ra_s \left( S/L \right) \right]^2} + \frac{2.87}{\left[ Ra_s \left( S/L \right) \right]^{1/2}} \right\}^{1/2}
\]

\[S_{opt} = 2.71 \left( \frac{Ra_s}{S^2 L} \right)^{-1/4}\]

\[S_{max} = 1.71 S_{opt}\]

\[
Nu_s = \left\{ \frac{48}{\left[ Ra_s^* \left( S/L \right) \right]^2} + \frac{2.51}{\left[ Ra_s^* \left( S/L \right) \right]^{2/5}} \right\}^{1/2}
\]

\[S_{opt} = 2.12 \left( \frac{Ra_s^*}{S^2 L} \right)^{-1/5}\]

\[S_{max} = 4.77 S_{opt}\]

\[
Nu_s = \left\{ \frac{144}{\left[ Ra_s \left( S/L \right) \right]^2} + \frac{2.87}{\left[ Ra_s \left( S/L \right) \right]^{1/2}} \right\}^{1/2}
\]

\[S_{opt} = 2.15 \left( \frac{Ra_s}{S^3 L} \right)^{-1/4}\]

\[S_{max} = 1.71 S_{opt}\]

\[
Nu_s = \left\{ \frac{24}{\left[ Ra_s^* \left( S/L \right) \right]^2} + \frac{2.51}{\left[ Ra_s^* \left( S/L \right) \right]^{2/5}} \right\}^{1/2}
\]

\[S_{opt} = 1.69 \left( \frac{Ra_s^*}{S^4 L} \right)^{-1/5}\]

\[S_{max} = 4.77 S_{opt}\]
12.10 Natural Convection in Enclosures

In all the natural convection applications discussed above, heat is exchanged between a surface and a surrounding fluid of infinite extent. In this section, we focus attention on natural convection inside enclosed spaces, where heat is transferred between two surfaces separated by a fluid. Applications occur in building design, double-pane windows, solar collectors, cryogenic containers, electronic equipment, and many other cases.

For example, consider the rectangular vertical enclosure shown in Figure S12-2. The length, $L$, is very large compared to the height, $H$, and spacing, $S$; therefore, we may approximate the geometry as two-dimensional as shown in Figure S12-2b. The left wall is at temperature $T_1$ and the right wall is at $T_2$, where $T_1 > T_2$. The top and bottom surfaces are perfectly insulated. Heat is transferred from the hot to the cold surface by natural convection of the fluid within the enclosure. The character of the buoyancy-induced flow depends on the Rayleigh number defined, for this case, as

$$Ra_s = Gr_s Pr = \frac{g \beta \rho^2 (T_1 - T_2) S}{\mu^2} Pr$$  

Eq. S12-1

If Rayleigh number is very low, ($Ra < 2000$), viscous forces are much larger than buoyancy forces and no fluid motion occurs. This can happen, for example, if the spacing, $S$, is small or the viscosity, $\mu$, is large. At these low Rayleigh numbers, heat transfer is by pure conduction in the fluid between the walls, and Nusselt number reduces to $Nu = 1$. At larger Rayleigh number, fluid near the hot left wall rises and fluid near the cold right wall sinks. A large rotating fluid cell promotes heat transfer between the plates. As Rayleigh number increases further, the boundary layers on right and left walls become thinner, and counter-rotating cells appear in the corners. At very high Rayleigh numbers, the flow becomes transient and turbulent. Table S12-2 gives correlations for Nusselt number in rectangular enclosures for various ranges of Rayleigh and Prandtl numbers. The Nusselt number and heat transfer rate equation to be used with these correlations is

$$Nu_s = \frac{hS}{k} \quad \dot{Q} = hA(T_1 - T_2) = hHL(T_1 - T_2)$$

Fluid properties are evaluated at the average of the plate temperatures, $T_1$ and $T_2$. 

Figure S12-2 Natural convection in a vertical enclosure
Table S12-2 Correlations for natural convection in vertical enclosures

<table>
<thead>
<tr>
<th>Fluid</th>
<th>$Ra_s$ Range</th>
<th>$Pr$ Range</th>
<th>$H/S$ Range</th>
<th>$Nu_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas</td>
<td>$&lt; 2 \times 10^3$</td>
<td>—</td>
<td>—</td>
<td>$Nu_s = 1$</td>
</tr>
<tr>
<td>Gas</td>
<td>$2 \times 10^3 - 2 \times 10^4$</td>
<td>$0.5 - 2$</td>
<td>$11 - 42$</td>
<td>$Nu_s = 0.197Ra_s^{0.25} (H/S)^{0.99}$</td>
</tr>
<tr>
<td>Gas</td>
<td>$2 \times 10^5 - 2 \times 10^7$</td>
<td>$0.5 - 2$</td>
<td>$11 - 42$</td>
<td>$Nu_s = 0.073Ra_s^{0.333} (H/S)^{0.99}$</td>
</tr>
<tr>
<td>Liquid</td>
<td>$&lt; 2 \times 10^3$</td>
<td>—</td>
<td>—</td>
<td>$Nu_s = 1$</td>
</tr>
<tr>
<td>Liquid</td>
<td>$10^4 - 10^7$</td>
<td>$1 - 20,000$</td>
<td>$10 - 40$</td>
<td>$Nu_s = 0.042Ra_s^{0.25} Pr^{0.012} (H/S)^{-0.3}$</td>
</tr>
<tr>
<td>Liquid</td>
<td>$10^8 - 10^9$</td>
<td>$1 - 20$</td>
<td>$1 - 40$</td>
<td>$Nu_s = 0.046Ra_s^{0.333}$</td>
</tr>
</tbody>
</table>

If the enclosure is horizontal, different flow patterns appear. Figure S12-3 shows a long horizontal enclosure with a hot bottom wall and a cold top wall. The vertical sides are perfectly insulated. For very low Rayleigh number ($Ra < 1700$), the fluid does not move, and heat transfer is by conduction only. At higher Rayleigh numbers, a series of well-ordered rotating eddies, called Benard cells, is induced in the flow, as shown in Figure S12-3. At very high Rayleigh numbers, the flow becomes disordered and turbulent. Table S12-3 gives correlations for horizontal enclosures for a range of Rayleigh and Prandtl numbers. In the case of a horizontal enclosure with a hot surface on top and a cold surface below, heat transfer is by conduction only and $Nu = 1$.

In the horizontal enclosure correlations, the Rayleigh number given in Eq. S12-1 is used. The Nusselt number and heat transfer rate are

$$Nu_s = \frac{hS}{k} \quad Q = hA(T_1 - T_2) = hWL(T_1 - T_2)$$

where $W$ and $L$ are the width and length of the enclosure, as shown in Figure S12-3. As before, properties are evaluated at the average of the two plate temperatures.

Some applications, such as solar collectors, involve a tilted enclosure, as shown in Figure S12-4. Correlations for vertical enclosures can be applied to tilted enclosures by replacing $g$ in the Rayleigh number with $g \cos \theta$. This modification is valid for $\theta < \theta_{crit}$. Values of $\theta_{crit}$ are given as a function of aspect ratio, $H/S$ in Table S12-4.
Figure S12-3  Natural convection in a horizontal enclosure

Table S12-3  Correlations for natural convection in horizontal enclosures

<table>
<thead>
<tr>
<th>Fluid</th>
<th>$Ra_s$ Range</th>
<th>$Pr$ Range</th>
<th>$Nu_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas</td>
<td>$&lt;1.7 \times 10^3$</td>
<td>—</td>
<td>$Nu_s = 1$</td>
</tr>
<tr>
<td>Gas</td>
<td>$1.7 \times 10^3 - 7 \times 10^3$</td>
<td>$0.5 - 2$</td>
<td>$Nu_s = 0.059 Ra_s^{0.4}$</td>
</tr>
<tr>
<td>Gas</td>
<td>$7 \times 10^3 - 3.2 \times 10^5$</td>
<td>$0.5 - 2$</td>
<td>$Nu_s = 0.212 Ra_s^{0.25}$</td>
</tr>
<tr>
<td>Gas</td>
<td>$&gt;3.2 \times 10^5$</td>
<td>$0.5 - 2$</td>
<td>$Nu_s = 0.061 Ra_s^{1/3}$</td>
</tr>
<tr>
<td>Liquid</td>
<td>$&lt;1.7 \times 10^3$</td>
<td>—</td>
<td>$Nu_s = 1$</td>
</tr>
<tr>
<td>Liquid</td>
<td>$1.7 \times 10^3 - 6 \times 10^3$</td>
<td>$1 - 5000$</td>
<td>$Nu_s = 0.012 Ra_s^{0.6}$</td>
</tr>
<tr>
<td>Liquid</td>
<td>$6 \times 10^3 - 3.7 \times 10^5$</td>
<td>$1 - 5000$</td>
<td>$Nu_s = 0.375 Ra_s^{0.2}$</td>
</tr>
<tr>
<td>Liquid</td>
<td>$3.7 \times 10^4 - 10^8$</td>
<td>$1 - 20$</td>
<td>$Nu_s = 0.13 Ra_s^{0.3}$</td>
</tr>
<tr>
<td>Liquid</td>
<td>$&gt; 10^8$</td>
<td>$1 - 20$</td>
<td>$Nu_s = 0.057 Ra_s^{1/3}$</td>
</tr>
</tbody>
</table>
Up until this point, we have discussed forced and natural convection as separate phenomena. However, forced convection is always accompanied by natural convection. The difference in temperature between surface and fluid that is present in forced convection produces buoyancy effects in the fluid. Under most practical circumstances, however, the influence of natural convection is negligible compared to forced convection.

Velocities induced by natural convection are typically small, and forced convection often overwhelms any natural convective effects. Exceptions occur if the forced flow is slow. To determine the relative importance of forced and natural convection, the Richardson number is defined as

$$ Ri = \frac{Gr}{Re^2} $$

The Richardson number represents the relative magnitude of natural to forced convection for a given fluid. If Richardson number is very low, then forced convection dominates; if it is very high, then natural convection dominates. When the Richardson number is close to unity, forced and natural convection are of comparable size and both should be considered. The range in which mixed forced and natural convective effects are important is

$$ 0.1 < Ri < 10 $$

Forced convection can either aid natural flow, oppose it, or be perpendicular to it. For example, consider a hot horizontal cylinder in cool air. The fluid in the natural convective boundary layer flows upward. If forced flow is also directed upward, then forced convection aids natural flow. If forced flow is directed downward over the cylinder, then it opposes natural convective flow. Finally, if forced flow is horizontal, it is perpendicular to natural convective flow.

The Nusselt number for mixed convection can be approximated by a relation of the form

![Figure S12-4 Natural convection in a tilted enclosure](image)

<table>
<thead>
<tr>
<th>H/S</th>
<th>1</th>
<th>3</th>
<th>6</th>
<th>12</th>
<th>&gt;12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{crit}$</td>
<td>65°</td>
<td>37°</td>
<td>30°</td>
<td>23°</td>
<td>20°</td>
</tr>
</tbody>
</table>

### 12.11 Mixed Forced and Natural Convection
\[ \textit{Nu}^n = \textit{Nu}_{\text{forced}}^n \pm \textit{Nu}_{\text{natural}}^n \]

where the Nusselt numbers for pure forced and pure natural convection are found from known correlations. The plus sign is used when forced flow is aiding or perpendicular and the minus sign is used when forced flow opposes natural flow. The value of \( n \) varies with geometry between 3 and 4. For vertical geometries, \( n = 3 \) is preferred, while for horizontal geometries, \( n = 3.5 \). For cylinders or spheres, use \( n = 4 \).