RAILPHS GROCERY COMPANY (http://www.ralphs.com), a subsidiary of The Kroger Company, operates over 300 supermarkets in southern California. Typically, such stores occupy 45,000 square feet and carry approximately 30,000 different items. Due to the large number of different goods carried by a typical supermarket, proper inventory control is essential to the profitability of the company.

Although some of the items sold in the supermarkets, such as Ralphs’ brand baked goods and dairy products are produced by the company in-house, the majority of the items sold in each supermarket, are goods purchased from other manufacturers. In some cases, manufacturers offer Ralphs discounts if it purchases over certain limits. Certain items have sufficiently long shelf lives that spoilage is typically not a factor. For other items, however (e.g., meat, produce, dairy products, periodicals), items not sold by a particular date must be either discounted or pulled from the shelves and disposed of.

Because the company has its own manufacturing facilities, it must also maintain adequate inventory to operate its machinery. While some of the maintenance is scheduled (and therefore necessary parts inventories can be anticipated), unplanned machine breakdowns requiring spare parts cannot be predicted.

Many of the items sold in the supermarket are stocked by Ralphs’ own employees. However, for some items (e.g., soft drinks, snack foods, and baked goods), the company relies on manufacturer’s representatives for stocking. In the former case, Ralphs frequently utilizes its automated checkouts system to determine when and how much to order of each item. In the latter case, deliveries of items are scheduled on a regular basis, and the manufacturer’s representatives determine the order quantities.
8.1 Overview of Inventory Issues

Proper control of inventory is crucial to the success of an enterprise. Profitability can suffer if a firm has either too much or too little inventory. Companies that have excess inventory are frequently forced to offer substantial mark-downs in order to dispose of this merchandise. This situation is especially prevalent in industries affected by periodic style changes, such as the automobile industry. New-car dealerships are much more willing to deal on price at the end of the model year than at the beginning.

Not having enough inventory can also lead to problems. A retail store that is frequently out of stock of popular items will soon lose its customers to the competition. A manufacturing firm that runs out of a crucial component may have to shut down its production lines, resulting in great expense and lost opportunities.

Managers often use inventory models to develop an optimal inventory policy, consisting of an order quantity denoted $Q$ and an inventory reorder point denoted $R$. Firms frequently have many thousands of different items known as stock-keeping units (SKUs) in their inventories. Ideally, a firm would like to determine the inventory policy for each SKU which minimizes its total variable costs over a given (possibly infinite) time horizon.

### Components of an Inventory Policy

- $Q = \text{Inventory Order Quantity}$
- $R = \text{Inventory Reorder Point}$

In inventory modeling, the costs associated with a particular inventory policy are assessed. If a firm orders in small amounts or produces in small batches, although the size (and cost) of the inventory may be relatively low, orders or production setups are more frequent, resulting in annual costs of ordering inventory or setting up production runs that are higher than those associated with larger quantities. If order or production sizes are larger, the number of orders or production runs and their associated fixed costs are less, but the costs associated with maintaining higher inventory levels are greater. Thus inventory analyses can be thought of as cost-control techniques that strike a balance between having too much and too little inventory.

### TYPES OF COSTS IN INVENTORY MODELS

The various costs in inventory models can be categorized into four broad areas: holding or carrying costs; order or setup costs; customer satisfaction costs, and procurement costs.

#### Holding Costs

**Holding**, or **carrying, costs** are those costs incurred by a firm to maintain its inventory position. The typical annual holding cost of a firm’s inventory is between 10% and 40% of the average inventory value. Given the high value of many firms' inventories, this rate can result in large annual expenditures. For example, the annual expense of a medium-sized lumberyard that has an inventory with an average value of a million dollars and a 30% annual holding cost rate is $300,000. If the lumberyard could cut this expense without affecting service, it could experience considerable savings.

Many factors affect a company’s holding cost rate, not the least of which is the cost of capital. Firms typically must borrow money in order to finance their inventory, and few firms are able to borrow at the prime rate. For small businesses, the cost of capital is typically prime plus 1 to 3%. These costs may be even higher if the firm has financed its expansion through the issuance of “junk” bonds or if it
lacks the creditworthiness necessary for standard banking relationships. Even if a firm has not had to borrow money to finance its inventory, it has foregone other investments that could have been made with available capital. Management must account for such opportunity costs when determining its holding cost rate.

Several other costs are associated with holding inventory. Since the product must be stored somewhere, the company must pay for rent, utilities, labor, insurance, and security of its inventory. In some localities, taxes must be paid based on the inventory’s value. Other costs include theft and breakage of inventory (which are classified under the more polite term shrinkage).

Another factor affecting inventory holding costs is deterioration or obsolescence. After a certain period of time, an item may lose some or all of its value. A car dealership left with last year’s models will have to reduce its prices. A supermarket left with sour milk on its shelves may have to pay to dispose of it.

All these costs are difficult, if not impossible, to measure. Thus the holding cost rate represents management’s best judgment of their total net effect. An important factor in determining the optimal inventory policy for an item is the cost of holding one unit of the item in inventory for a full year. When the holding cost rate of an item is known, its annual holding cost per unit, $C_h$, can be calculated by multiplying the annual holding cost rate, $H$, of an item by its unit cost, $C$.

**Holding Costs**

| $C_h$ = Annual Holding Cost per Unit in Inventory (in $ per unit in inventory per year) |
| $H$ = Annual Holding Cost Rate (in % per year) |
| $C$ = Unit Cost of an Item (in $ per item) |
| Thus, $C_h = H \times C$ |

**Order/Setup Costs**

Order costs are incurred when a firm purchases goods from a supplier. These can include postage, telephone charges, the expense to write up or phone in an order, the cost to check the order when it is received, and other fixed labor and transportation expenses that do not depend on the order size.

If the firm produces goods for sale to others, a production setup cost is normally incurred. This is the expense associated with beginning production of a particular item. For example, an ice cream manufacturer making a variety of flavors must clean out the machinery before it can begin producing a new flavor; machines producing ball bearings may need to be recalibrated when a new size is produced; and the staff at an aircraft manufacturer may need some refresher training prior to beginning a new production run.

The cost of placing an order or arranging a production setup, $C_o$, is independent of the order or production quantity. Because this cost is principally labor related, it can usually be readily measured.

**Order or Setup Costs**

| $C_o$ = Order or Production Setup Cost (in $ per order or $ per setup) |

**Customer Satisfaction Costs**

One possible inventory policy is to stock no inventory. Customers desiring the product simply place an order and wait for its arrival. In this case, customer satisfaction is likely to be lower than it would be if the item were readily available. Low
customer satisfaction may result in declining revenue and profitability. **Customer satisfaction costs** measure the degree to which a customer is satisfied with the firm’s inventory policy and the impact this has on long-term profitability.

In some cases, customer satisfaction costs are relatively easy to quantify. For example, if a retailer is out of stock of an item, the customer may be offered a more expensive substitute at the same price or a discount on the item if the customer is willing to wait. But what about a customer who is unwilling to wait, goes elsewhere for the item, and decides to stick with the competition for future purchases? It is extremely difficult to estimate the satisfaction cost in these cases.

For a customer who encounters an out-of-stock inventory situation and is willing to wait for the item, customer satisfaction costs can have both a fixed and a variable component. The fixed component, $C_b$, consists of costs that are independent of the length of time a customer must wait for an item, such as the administrative costs of issuing a “rain check,” recording the order, and contacting the customer when the merchandise arrives. The variable component, $C_s$, is a function of the length of time the customer must wait for goods to become available. Typically, the longer a customer waits for the item to arrive, the less satisfied the customer will be. The calculation of the variable component may be straightforward (such as when a firm offers a discount for each week a customer must wait for the item). More often, however, the variable component represents a goodwill cost that may be difficult to estimate. In such instances, focus groups can be used to obtain an estimate of this cost.

![Customer Satisfaction Costs]

- $C_b = \text{the fixed administrative cost of an out-of-stock item (in $ per stockout unit)}$
- $C_s = \text{the annualized cost of a customer waiting for an out-of-stock item (in $ per item out of stock per year)}$

**Procurement/Manufacturing Costs**

**Procurement** or **manufacturing costs** represent the cost of the items placed in inventory. If the item is obtained from an outside supplier, the procurement cost is the purchase cost per unit together with any shipping costs paid on a per unit basis. In some cases, a vendor may offer quantity discounts that enable the buyer to pay a reduced cost per unit if the amount purchased is above certain thresholds. In these instances, the order quantity plays an important role in determining the procurement cost. If the item is manufactured in-house, the procurement cost represents the incremental production cost per unit. Note that the procurement cost in this case does not include the production setup cost.

Table 8.1 summarizes these various inventory costs.

<table>
<thead>
<tr>
<th>Table 8.1 Inventory Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Holding Costs</strong> ($C_h$)</td>
</tr>
<tr>
<td>Cost of capital</td>
</tr>
<tr>
<td>Rent</td>
</tr>
<tr>
<td>Utilities</td>
</tr>
<tr>
<td>Insurance</td>
</tr>
<tr>
<td>Labor</td>
</tr>
<tr>
<td>Taxes</td>
</tr>
<tr>
<td>Shrinkage</td>
</tr>
<tr>
<td>Spoilage</td>
</tr>
<tr>
<td>Obsolescence</td>
</tr>
</tbody>
</table>
DEMAND IN INVENTORY MODELS
A key component affecting an inventory policy is the demand rate for a stock-keeping unit. Although future demand is generally not known with certainty, forecasting techniques (see Chapter 7) can generally provide good estimates for these values. Demand can be estimated for any future period; however, we will typically utilize the annual demand in developing our inventory models.

Perhaps the strongest factor influencing how we model a particular inventory situation is the demand pattern for the SKU in question. Demand that is projected to be reasonably constant over time must be modeled differently than demand that is highly variable. In this chapter, we limit our investigation to situations in which demand occurs at a known annual constant rate, D.

**Demand in Inventory Models**

\[ D = \text{Estimate of the Annual Demand for the Stock-Keeping Unit} \]

INVENTORY CLASSIFICATIONS
Inventory can be classified in various ways, depending on the issues that concern management. Table 8.2 summarizes different inventory classifications.

<table>
<thead>
<tr>
<th>By Process</th>
<th>By Importance</th>
<th>By Life</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw materials</td>
<td>A,B,C</td>
<td>Perishable</td>
</tr>
<tr>
<td>Work in progress</td>
<td></td>
<td>Nonperishable</td>
</tr>
<tr>
<td>Finished goods</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Process Classification**
Accountants at manufacturing firms typically classify inventory into three categories as defined by the production process: raw materials, work in progress, and finished goods. This categorization enables management to track the production process and determine whether it has adequate inventory levels to support the projected demand. Financial analysts use such information to detect any changes in a firm’s operations that might distort its profitability.

**A,B,C Classification**
A second way of classifying inventory is by the relative importance of the stock-keeping unit in terms of the firm’s capital needs. For example, the annual inventory value for each stock-keeping unit is determined by multiplying its unit cost by its annual demand. Normally, only 5% to 10% of the SKUs account for about 50% of a company’s total inventory value. These SKUs are classified as A units. Another 40% to 50% of the SKUs account for all but a small percentage of the firm’s total inventory value. These are classified as B units. The remaining nearly 50% of the SKUs usually account for a small percentage (less than 5%) of the firm’s total inventory value. These are classified as C units.

This A,B,C classification is useful in determining how much attention should be given to each stock-keeping unit in determining an inventory policy. A items are more carefully analyzed than B items. Because of the cost of inventory control, little, if any, analysis is done for C items.
Shelf Life Classification

Inventory models can also be classified by the shelf life of the inventory units. Certain perishable items, such as dairy products, baked goods, and periodicals, have a very short shelf life (no one wants to buy yesterday’s news). Management of these inventory items is quite different than for items that can remain in inventory for long periods of time with no noticeable deterioration of quality.

REVIEW SYSTEMS

Two types of review systems are widely used in business and industry for controlling stock-keeping units. In a continuous review system, the inventory is constantly monitored, and a new order is placed when the inventory level reaches a certain critical point. In a periodic review system, the inventory position is investigated on a regular basis (once a day, twice a week, etc.), and orders are placed only at these times. Both systems are discussed in this chapter.

8.2 Economic Order Quantity Model

ASSUMPTIONS OF THE EOQ MODEL

One of the most commonly used techniques for inventory optimization is the economic order quantity (EOQ) model. This model is useful for analyzing stock-keeping units that meet the following criteria:

- Demand for the item occurs at a known and reasonably constant rate.
- The item has a sufficiently long shelf life (i.e., there is little or no spoilage).
- The item is monitored using a continuous review system.

The EOQ model assumes that all parameters, including demand, remain constant forever—that is, over an infinite time horizon. Although no business will carry a stock-keeping unit indefinitely, using an infinite time horizon avoids the need to specify just how long the item will be stocked.

The lead time, \( L \), for an order represents the time that elapses between placement of an order and its actual arrival. In a basic EOQ model, we initially make the (unrealistic) assumption that \( L = 0 \). (This assumption is modified later.) Under these circumstances, it does not pay to order additional items until the exact instant we run out of stock. Furthermore, because demand is assumed to be constant over an infinite time horizon, whenever the firm runs out of inventory, it faces exactly the same future demand pattern as the previous time it ran out of inventory. For this model, a stationary inventory policy that orders the same amount each time must be optimal.

COST EQUATION FOR THE EOQ MODEL

Figure 8.1 illustrates the inventory profile of ordering \( Q \) units each time. In this figure, the horizontal axis represents time, and the vertical axis represents the inventory level. The time between orders, \( T \), is the cycle time. To build an EOQ model, we need to know the demand forecast, \( D \), for a given period (typically one year), the unit holding cost, \( C_h \), over the same period, and the ordering cost, \( C_o \). We can then construct an equation for the total annual inventory cost consisting of annual holding costs, ordering costs, and procurement costs, expressed in terms of the order quantity, \( Q \).
CHAPTER 8 Inventory Models

Total Annual Holding Costs

As Figure 8.1 illustrates, the inventory level for each cycle begins at level Q when the order arrives and is depleted at a constant rate to 0 just prior to the next order’s arrival. Due to this constant demand rate, the average inventory level over time is Q/2. Thus the annual holding cost associated with the policy of ordering Q units can be modeled as follows:

\[
\text{Total Annual Holding Cost} = \frac{Q}{2} \times \text{Annual Holding Cost per Unit}
\]

Total Annual Ordering Costs

If the annual demand is D and the order quantity is Q, the number of orders placed during the year is D/Q. Since the cost of placing each order is Co, the total annual ordering cost is as follows:

\[
\text{Total Annual Ordering Cost} = \frac{D}{Q} \times \text{Cost to Place an Order}
\]

Total Annual Procurement Costs

Since we seek to satisfy demand, we will purchase D items during the year at a cost of $C each. Thus the total annual item costs can be expressed as follows:

\[
\text{Total Annual Procurement Cost} = D \times \text{Cost per Unit}
\]
Total Annual Inventory Costs and the EOQ Formula

Because the lead time for delivery of an order is assumed to be 0, there will never be any shortage costs. Thus the total annual inventory costs, TC(Q), can be expressed by:

\[
TC(Q) = \left( \frac{Q}{2} \right) C_h + \left( \frac{D}{Q} \right) C_o + DC
\]  

or

\[
TC(Q) = \left( \frac{Q}{2} \right) C_h + \left( \frac{D}{Q} \right) C_o + \frac{DC}{Q^*}
\]  

Here, since DC is a constant, we can represent the total annual variable costs dependent on Q; by

\[
TV(Q) = \left( \frac{Q}{2} \right) C_h + \left( \frac{D}{Q} \right) C_o
\]

* This equation is modified in Equation 8.6 to account for costs related to carrying safety stock.

We can find \( Q^* \), the value of Q that minimizes \( TV(Q) \) in Eq. (8.2) (and hence \( TC(Q) \) in Equation 8.1) using calculus (see Appendix 8.2 on the accompanying CD-ROM). The following relationship, known as the EOQ formula, gives the value for \( Q^* \).

\[
Q^* = \sqrt{\frac{2DC}{C_h}}
\]

SENSITIVITY ANALYSIS IN EOQ MODELS

Figure 8.2 shows a typical graph of \( TV(Q) \) versus Q. As you can see, at the point \( Q^* \) where \( TV(Q) \) is minimized, the total annual holding and ordering costs are equal. The cost curve is also reasonably flat around \( Q^* \). Thus small variations in \( Q^* \) will not greatly affect total annual variable inventory costs. This is important because the optimal order quantity \( Q^* \) is generally rounded off to an integer value or further modified owing to shipping restrictions. As long as these differences are not too large, the total annual variable cost will not be greatly affected.

Another factor that might affect the calculation of \( Q^* \) is the variability in actual demand. If demand varies from the estimated forecast by a modest amount, however, the error in \( Q^* \) is small and there is only a minor increase over the optimal cost. Accordingly, demand variability when calculating \( Q^* \) for an EOQ model is disregarded and it is assumed that the annual demand is constant and known with certainty.
**CYCLE TIME/NUMBER OF ORDERS PER YEAR**

The cycle time, $T$, represents the time that elapses between the placement of orders. It can be calculated by dividing the order quantity by the annual demand, that is, $T = \frac{Q}{D}$. Since the cycle time corresponds to the age of the last item sold from inventory, it can be compared to the shelf life to determine if items will go bad while in inventory. If the cycle time is greater than the shelf life, the model must be modified. The reciprocal of this quantity, $\frac{D}{Q}$, gives the average number of orders per year, $N$.

**LEAD TIME AND THE REORDER POINT**

In the preceding analysis, it was assumed that the lead time, $L = 0$. In reality, lead time is always positive and must be accounted for when deciding when to place an order. The reorder point, $R$, is the inventory position of the item when an order is placed.

To determine the reorder point, we note that, since demand is constant at rate $D$, the total demand during lead time, $L$, is simply $L \cdot D$, where $L$ and $D$ are expressed in the same time units (years, weeks, days, etc.). Hence, if an order is placed when the inventory level is at $L \cdot D$, the order will arrive precisely when the inventory level is at $0$. This gives the inventory profile shown in Figure 8.1.

$$R = L \cdot D$$ \hspace{1cm} (8.4)

In some instances, the lead time may be of sufficient length that it exceeds the cycle time. In such cases, since $L > \frac{Q}{D}$, $L \cdot D$ will exceed $Q$, and it will be impossible to order the item for delivery during its current inventory cycle. Hence, this order would have to be placed during a previous cycle, and, as shown in Figure 8.3, there will be times when more than one order is outstanding. A company’s stock on hand plus the size of any outstanding orders not yet delivered is known as its inventory position. At the reorder point, its inventory position is $L \cdot D$. 

---

**FIGURE 8.2** Total Annual Inventory Holding and Ordering Cost

- **Cycle Time/Number of Orders per Year**
  - Cycle Time: $T = \frac{Q}{D}$ (in years)
  - Number of Orders per Year: $N = \frac{D}{Q} = \frac{1}{T}$ (in orders per year)
Safety Stock

This calculation of the reorder point assumed that there is constant demand for the SKU and that there is a fixed lead time. In reality, demand usually fluctuates, and lead time varies. To account for these variations, most firms build a safety stock (SS) into their inventory policy. The safety stock acts as a buffer to handle higher than average lead time demand or longer than expected lead time. Including the safety stock, the reorder point is expressed by the following formula:

\[
R = (\text{Lead Time}) \times (\text{Demand Rate}) + \text{Safety Stock}
\]

\[
R = L \times D + SS \tag{8.5}
\]

L and D must be expressed in the same time units.

Carrying safety stock, however, increases the annual holding costs and hence the total annual cost, TC(Q), by an amount equal to \( C_h \) times SS.

\[
TC(Q) = \left( \frac{Q}{2} \right) C_a + \left( \frac{D}{Q} \right) C_p + DC + C_h SS = TV(Q) + DC + C_h SS \tag{8.6}
\]

The determination of the amount of safety stock that a firm should carry is often based on a desired service level approach. In Section 8.3 we show how one can determine the appropriate amount of safety stock based on the desired likelihood of encountering an out-of-stock situation.

To illustrate the concepts, consider the case of the Allen Appliance Company.

ALLEN APPLIANCE COMPANY

Allen Appliance Company (AAC) wholesales small appliances—toasters, mixers, blenders, and so on—to 90 retailers throughout Texas. One of its products, the Citron brand juicer, has shown a gradual decline in sales over the past several years. While this decline may be attributed to several factors, such as increased
cost of fresh oranges, better availability of “fresh-squeezed” juice at grocery stores, or less time available to make juice by hand, the bottom line is that sales have fallen to a rate that is lower than in previous years.

Several years ago, Mr. Allen hired a consultant who recommended an inventory policy of ordering 600 juicers whenever the inventory level reached 205. Although Mr. Allen has religiously followed this policy in the past, he wonders whether, given the reduction in sales, this policy is still optimal.

The juicers cost AAC $10 each and are sold to customers for $11.85 each. Mr. Allen is able to borrow money at a 10% annual interest rate. He estimates that storage and other miscellaneous costs amount to about 4% of the average inventory value per year.

Based on labor, postage, and telephone charges, Mr. Allen estimates that it costs $8 to place an order with the Citron Company. It takes a worker who earns $12 per hour 20 minutes to check the shipment when it arrives. AAC is open five days a week, 52 weeks a year. Lead time is approximately eight working days, and AAC monitors its inventory using a continuous review system. Over the past 10 weeks, demand for the juicers has been as shown in Table 8.3. Given this information, Mr. Allen wishes to know if he should revise his current inventory policy.

### Table 8.3
Sales of Juicers Over Previous 10 Weeks

<table>
<thead>
<tr>
<th>Week</th>
<th>Sales</th>
<th>Week</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>105</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>115</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>125</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>120</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>125</td>
<td>10</td>
</tr>
</tbody>
</table>

**SOLUTION**

In order to select the appropriate inventory model to analyze this situation, we must first investigate the demand pattern for the juicers. Weekly sales, though not exactly constant, do not vary greatly. Hence, we will analyze this situation using the economic order quantity model.1

To determine a representative value for weekly demand, we can use one of the various time series techniques appropriate for stationary models discussed in Chapter 7. After consulting with other members of the management science team, suppose we select a 10-week simple moving average approach to forecast future weekly demand. By averaging demand over the most recent 10 weeks, we forecast a weekly demand of \((105 + 115 + \ldots + 130)/10 = 120\).

In an EOQ analysis, it is important to express all data values in the same time units. Mr. Allen collects data on an annual basis; therefore he needs a forecast of annual demand for the juicer. Because he feels reasonably confident that demand for the juicers has bottomed out, our forecast for the annual demand is \((120)(52) = 6240\) units.

The annual holding cost rate for the juicers consists of the sum of the annual interest rate (10%) and the annual storage and miscellaneous costs (4%). Hence, using a holding cost rate of 10% + 4% = 14%, the annual holding cost of a juicer left in inventory for an entire year, \(C_h\), is \((10)(14) = $14.00\). Note that we use AAC’s cost, not its selling price, to determine the annual holding cost per unit.

---

1 In some situations, the only historic data a firm has are sales data. If sales patterns exhibit erratic behavior, underlying demand could be constant but the item might be frequently out of stock.
Mr. Allen’s ordering cost consists of the $8 cost of placing the order and the cost involved in checking the order upon its arrival. Since checking the shipment requires 20 minutes, and checkers make $12 per hour, the checking costs are \((20/60)(12) = $4\) per order. Hence, the total ordering cost, \(C_o\), is \(8 + 4 = $12\).

In summary, the following data exist for this problem:

- \(D = 6240\)
- \(C = $10\)
- \(H = 14\%\)
- \(C_h = $1.40\)
- \(C_o = $12\)

**Analysis of Current Policy**

Mr. Allen’s current policy is to order 600 juicers at a time, yielding an annual holding cost of \((600/2)(1.40) = $420\) and an annual order cost of \((6240/600)(12.00) = $124.80\). Thus the total annual variable cost of this policy is:

\[
TV(600) = \left(\frac{600}{2}\right)(1.40) + \left(\frac{6240}{600}\right)(12) = $544.80
\]

Since AAC is open five days a week, the average weekly demand of 120 juicers translates into an average demand of \(120/5 = 24\) juicers per working day. Given a lead time of eight working days, the reorder point without safety stock should be equal to \(L = D = 8 \times 24 = 192\). Since AAC is using a reorder point of 205, we can infer that it desires to have \(205 - 192 = 13\) units of safety stock. Thus from Equation 8.6 we see that AAC’s total annual cost based on its current inventory policy is:

\[
TC(600) = 544.80 + 6240(10) + 13(1.40) = $62,963.00
\]

**Analysis of Optimal Policy with Safety Stock = 13**

If we use the EOQ formula to determine the order quantity, we get:

\[
Q^* = \sqrt{\frac{2(6240)(12)}{1.40}} = 327.065
\]

Since Mr. Allen cannot order .065 of a juicer, we round this answer to an order quantity of 327.

Substituting \(Q^* = 327\) and \(SS = 13\) into Equations 8.2 and 8.6 gives:

\[
TV(327) = \left(\frac{327}{2}\right)(1.40) + \left(\frac{6240}{327}\right)(12) = $457.89
\]

and

\[
TC(327) = 457.89 + 6240(10) + 13(1.40) = $62,876.09
\]

Thus adopting this policy will result in an annual savings of \(544.80 - 457.89 = $86.91\) in variable costs. This savings might seem small in absolute dollars, but on a percentage basis, it amounts to approximately 16% of the current annual variable costs for this one SKU. To put this in perspective, if AAC carries 2000 different SKUs in inventory and has total annual variable inventory costs of a quarter of a million dollars, a 16% savings in inventory holding and ordering costs for each SKU translates into approximately a $40,000 annual savings!

---

1 This calculation is based on \(L\) and \(D\) expressed in days. Expressed in years, \(L = 8/260 = .03077\) and \(R = (.03077)(6240) = 192\).
If AAC uses an order quantity of $Q^* = 327$, the cycle time is:

$$T = \frac{Q^*}{D} = \frac{327}{6240} = 0.0524 \text{ year}$$

$$= (0.0524 \text{ year})(52 \text{ weeks/year})(5 \text{ days/week}) \approx 14 \text{ working days}$$

These calculations indicate that the juicers will be sold in a reasonably short period of time after they enter AAC’s inventory. Thus shelf life is not a factor. Since $T \approx 14$ working days, AAC will place orders for the juicers approximately every two and three-quarter weeks. This information can be useful if Mr. Allen decides to coordinate orders for other items from Citron along with the juicers.

**SENSITIVITY OF THE EOQ RESULTS—LOTS OF 100**

Suppose Citron’s policy requires AAC to purchase juicers in units of 100. In this case, AAC could not order 327 juicers and would modify its order quantity to, say, 300. The total annual inventory holding and ordering cost of a policy based on orders of 300 juicers at a time amounts to $459.60, an increase of only $1.71 (or less than one-half of 1% of the total variable costs) per year over the cost of ordering 327 juicers at a time. This cost increase is so slight that an order quantity of 300 might even be preferable just for the sake of operational convenience.

**EFFECT OF CHANGES IN INPUT PARAMETERS**

One of the properties of the EOQ model is that the optimal total cost is relatively insensitive to small or even moderate changes to one of the input parameters of the model. To illustrate, suppose that, due to a “back to nature” craze, the actual annual demand for juicers turns out to be 7500 instead of the forecasted 6240 (an increase of over 20%). Using the EOQ formula, we find that AAC should have ordered $Q^* = 359$ juicers at a time rather than the 327 we calculated. The annual holding and ordering cost (excluding safety stock costs) associated with an annual demand of 7500 juicers and an order quantity of 359 is:

$$TV(359) = \left(\frac{359}{2}\right)(1.40) + \left(\frac{7500}{359}\right)(12) = 502.00$$

If instead of ordering 359 juicers we used the order quantity of 327, the annual ordering and holding cost would be

$$TV(327) = \left(\frac{327}{2}\right)(1.40) + \left(\frac{7500}{327}\right)(12) = 504.13$$

This is an increase of only $2.13, or 0.4% per year! Thus a “mistake” of more than 20% in estimating demand has less than a 0.4% effect on the total annual variable cost.

**Software Results**

Figure 8.4 gives an Excel spreadsheet that can be used to determine the optimal order quantity and reorder point as well as the inventory costs associated with any specified order quantity. The input parameters are entered in column B. (Note that the value for Ch in cell B13 can be determined by multiplying the value in cell B11, the cost per
unit, by the value in cell B12, the annual holding cost rate.) The outputs in column E correspond to the optimal order quantity, \( Q^* \), whereas the outputs in column H are for a predetermined order quantity that is put in cell H10 (in this case \( Q = 600 \)).

While the formulas shown in Figure 8.4 are generally straightforward and follow the algebraic formulas presented above, the formula for the reorder point, \( R \), in cell E13 is somewhat complex. This is because the formula must allow for the possibility that the lead time is so long that multiple orders may be outstanding. The portion of the formula, 

\[-\text{INT}\left(\frac{\text{INT}\left(\frac{B15}{600}\right) + C10}{E10}\right)\frac{B15}{600},\] 

accounts for this possibility.

**USING SOLVER TO DETERMINE \( Q^* \)**

It is also possible to get the value of \( Q^* \) by using the Solver option in Excel. To do this with the spreadsheet shown in Figure 8.4, click on Solver and in the “Set Target Cell” box put $H14$, in the “Equal To” section, highlight the Min button, and in the “By Changing Cells” box put $H10$. Then, click Solve. The EOQ solution will appear in cell H10 and the values in column H will be identical to those in column E.

**USING THE INVENTORY.XLS TEMPLATE FOR SOLVING THE EOQ MODEL**

As an alternative to constructing an Excel spreadsheet to solve EOQ problems, we have included the template `inventory.xls` on the accompanying CD-ROM. This template will enable you to quickly solve all of the inventory models discussed in this chapter without having to construct spreadsheets from scratch. The worksheet `EOQ` contained in the template (which is similar in form to the spreadsheet shown in Figure 8.4), can be used to solve EOQ problems. Details on using the template are contained in Appendix 8.1.
8.3 Determining Safety Stock Levels

As we discussed in Section 8.2, when businesses want to avoid stockouts, they incorporate safety stock requirements to determine the reorder point. One approach used to determine the appropriate safety stock level for an item is for management to specify a desired service level. This can represent one of two quantities:

1. the likelihood or probability of not incurring a stockout during an inventory cycle, or
2. the percentage of demands that are filled without incurring any delay.

The first approach, known as the cycle service level, is appropriate when the firm is concerned about the likelihood of a stockout and not its magnitude. It is used, for example, in manufacturing settings in which any stockout affects production.

The second approach, known as the unit service level, is appropriate when the firm is interested in controlling the percentage of unsatisfied demands. It corresponds to the term fill rate and is what managers commonly mean when they state a service level.

To illustrate the difference between these two service levels, consider again the Allen Appliance Company (AAC) example described in Section 8.2. Table 8.4 shows the juicer demand and number of units on backorder during the last five inventory cycles at AAC when the policy was to order 600 units whenever the inventory level reached 205 units. We see that, during this time period, the cycle service level is 80% since four out of five cycles experienced no stockouts. At the same time, the unit service level is 99.5% since only 15 of 3000 units were backordered.

<table>
<thead>
<tr>
<th>Cycle Number</th>
<th>Demand</th>
<th>Number of Units on Backorder</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>585</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>610</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>628</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>572</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>605</td>
<td>0</td>
</tr>
</tbody>
</table>

THE CYCLE SERVICE LEVEL APPROACH

Stockouts occur only if demand during the lead time exceeds the reorder point. For example, if AAC uses a reorder point of 205, a stockout occurs only when demand during the lead time exceeds 205 units. Hence, if the lead time demand distribution is known, a statistical analysis can be used to determine the service level corresponding to a given reorder point or the safety stock required to maintain a given cycle service level.

In many cases, long-run demand can be assumed to be relatively constant, even when for shorter intervals, demand can be more appropriately modeled by a normal distribution. Thus demand during the lead time would be modeled by a normal distribution with estimated mean, \( \mu_L = L \cdot D \), and standard deviation, \( \sigma_L \).

From basic statistics we know that in this case the demand level that has a probability of \( \alpha \) of being exceeded is given by the formula: \( \mu_L + z_\alpha \sigma_L \). Here \( z_\alpha \) corresponds to the \( z \) value that puts probability \( \alpha \) in the upper tail of the normal distribution. (The values of \( z \) are found in Appendix A.) Hence, since a \( (1 - \alpha) \) service level corresponds to a probability of stockout equal to \( \alpha \), the reorder point is \( \mu_L + z_\alpha \sigma_L \). The safety stock, SS, is therefore equal to \( z_\alpha \sigma_L \).
For example, recall that the lead time for delivery of juicers at AAC is eight working days and that demand over the past 10 weeks is that given in Table 8.3. From these data, the sample mean and variance, which can be used as estimates for the population mean and variance for weekly demand is calculated by:

\[
\mu \approx \bar{x} = \frac{(105 + 115 + \ldots + 130)}{10} = 120
\]

\[
\sigma^2 \approx s^2 = \frac{(105^2 + 115^2 + \ldots + 130^2) - 10(120^2)}{9} = 83.33
\]

The estimates of the mean, \( \mu_L \), and variance, \( \sigma_L^2 \), of the lead time demand are then found by multiplying the weekly demand estimates by the length of the lead time (expressed in weeks). Since the lead time is eight days, or \( 8/5 = 1.6 \) weeks, these values are:

\[
\mu_L \approx 1.6(120) = 192
\]

\[
\sigma_L^2 \approx 1.6(83.33) = 133.33
\]

Thus

\[
\sigma_L \approx \sqrt{133.33} = 11.55
\]

**Analysis of Current Policy**

If it can be assumed that lead time demand follows a normal distribution, a reorder point of \( R = 205 \) implies:

\[
205 = 192 + z(11.55)
\]

Solving for \( z \) gives:

\[
z = (205 - 192)/11.55 = 1.13
\]

We see from Appendix A that the area in the tail above \( z \) is \( \alpha = 1.13 \) is \( .5 - .3708 = .1292 \approx .13 \). Thus, as shown in Figure 8.5, this policy corresponds approximately to an 87% cycle service level.

---

**FIGURE 8.5** Determining a Cycle Service Level for AAC

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\( ^1 \) This can be verified by performing a standard goodness-of-fit test as described in Appendix 9.2 on the accompanying CD-ROM.
Determining the Reorder Point for a Given Cycle Service Level

Now suppose that AAC management wants to improve its cycle service level from 87% to 99%. In this case, $\alpha = .01$ and $z_{.01} = 2.33$. As illustrated in Figure 8.6, the reorder point is $R = 192 + 2.33 \times 11.55 = 219$ units. The corresponding safety stock is then $2.33 \times 11.55 \approx 27$ units.

![Figure 8.6 Reorder Point for AAC Corresponding to a 99% Cycle Service Level](image)

Determining the Reorder Point for a Given Acceptable Number of Stockouts per Year

Another way of expressing a cycle service level is to specify an acceptable value for the likelihood of being out of stock for a given average number of cycles per year. For example, suppose AAC is willing to run out of juicers an average of at most one cycle per year. If the firm uses an order quantity of 327 units, the average number of cycles per year is $N = \frac{6240}{327} = 19.08$. Thus the acceptable probability of a stockout during a lead time is $\alpha = 1/19.08 = .0524$, which corresponds to a 94.76% service level. Referring to Appendix A, this gives us $z_{.0524} \approx 1.62$. In this case, the safety stock, $z_{.0524} \approx 1.62(11.55) \approx 19$ and the reorder point would be $R = 192 + 19 = 211$ units.

Software Results

Figure 8.7 shows an Excel spreadsheet that determines the reorder point for a given cycle service level or the cycle service level for a given reorder point. The appropriate values are entered in column B. (Normally one would specify either the desired service level in cell B7 or the reorder point in cell B8.) The assumption made for lead time demand in this spreadsheet is that it follows a normal distribution.

![Figure 8.7 Excel Spreadsheet for Allen Appliance Company Reorder Point](image)
From this spreadsheet we see that if the desired service level is 99% (99 in cell B7), the reorder point should be approximately 219 (the value in cell E5). Alternatively, if the reorder point is 205 (the value in cell B8), the corresponding cycle service level is approximately 87% (the value in cell E6).

The Excel template inventory.xls on the accompanying CD-ROM contains a worksheet Cycle Service that can be used for determining the reorder point using a cycle service level approach. The format for the worksheet is similar to the spreadsheet shown in Figure 8.7.

**THE UNIT SERVICE LEVEL APPROACH**

Finding the reorder point corresponding to a desired unit service level is a bit more complicated than finding the reorder point corresponding to a desired cycle service level. Appendix 8.3 on the accompanying CD-ROM contains the general formula used to find this value. When lead time demand follows a normal distribution with estimated mean \( \mu_L \) and standard deviation \( \sigma_L \), the reorder point, \( R \), can be determined as follows:

1. Determine the value of \( z \) that satisfies the relationship:

\[
L(z) = \frac{(1 - \text{service level})Q^*}{\sigma_L} = \frac{\alpha Q^*}{\sigma_L}
\]

(Here \( L(z) \) represents the partial expected value for the standard normal random variable between some value \( z \) and infinity. Values of the \( L(z) \) function are given in Appendix B.)

2. Solve for \( R \) using the formula \( R = \mu_L + z\sigma_L \).

For example, suppose AAC desires a 99% unit service level. Based on values \( \mu_L = 192 \), \( \sigma_L = 11.55 \), and \( Q^* = 327 \), for a 99% unit service level, \( L(z) = (0.01)(327/11.55) = 0.2831 \). Referring to Appendix B, we see that a value of \( L(z) = 0.2831 \) corresponds to \( z = 0.26 \). Therefore, \( R = 192 + 0.26(11.55) = 195 \) and the safety stock = 0.26(11.55) = 3 units.

It is interesting to compare the safety stock requirements between a cycle and a unit service level. For the Allen Appliance example, the safety stock equals only three units for a 99% unit service level, compared to 27 units for a 99% cycle service level. The safety stock requirements are lower for the unit service level because the calculations include the nonlead time portions of the inventory cycle. Since it is impossible to be in an out-of-stock situation during these times, fewer safety stock units are necessary. Another way to view this is to recognize that, when we select a reorder point based on a desired unit service level, the corresponding cycle service level will be lower.

Worksheet Unit Service contained in the Excel spreadsheet inventory.xls on the accompanying CD-ROM is a template for determining the reorder point using a unit service level approach.

**MANAGEMENT REPORT**

On the basis of the analysis of the last two sections, the following memorandum was prepared. In this report, we outline an optimal inventory policy for AAC and examine the sensitivity of these recommendations to changes in the annual demand and holding cost.
MEMORANDUM

To: Mr. James P. Allen, President—Allen Appliance Company
From: Student Consulting Group
Subj: Inventory Policy for Citron Juicers

Due to a gradual erosion in demand over the past several years, we have been asked to analyze the current inventory policy for Citron juicers and make policy recommendations that might help Allen Appliance Company lower its inventory costs. We have analyzed the inventory situation for these juicers and are pleased to report our findings.

Our analysis indicates that demand for the juicers is fairly constant and that the products have a shelf life in excess of three months. Based on the past 10 weeks of sales, we forecast the annual demand for juicers to be 6240 units. We assume that AAC will continue to operate five days a week, that lead time for delivery is estimated to be eight working days, and that the juicers must be ordered in multiples of 100 units from Citron.

The following cost data have been provided by Allen management and used in our analysis:

- Unit cost per juicer: $10
- Ordering cost: $12
- Annual holding cost rate: 14%

Based on our analysis of this data we recommend the following:

1. Lower the order quantity for Citron juicers from 600 to 300 units. This should result in a reduction in the annual holding and ordering costs from $544.80 to $459.60 (a savings of $85.20, or 15.6% per year).
2. Set the reorder point for the juicers at 219, including a safety stock of 27 units. Although changing the reorder point from 205 to 219 increases the annual safety stock holding costs by $20, this additional cost should be more than offset by the greater customer satisfaction generated from having fewer customers finding an out-of-stock situation.

Figure 1 compares the current and proposed policies. While we are confident of the ordering cost data, management has indicated some uncertainty regarding both the value of $1.40 used for the annual holding cost per juicer and our forecast of 6240 for the annual juicer demand. We have therefore considered different annual holding cost and demand amounts and examined the savings in total annual variable inventory costs of using our recommended policy versus AAC’s current policy. Table I gives these percentage savings.
8.4 EOQ Models with Quantity Discounts

Quantity discounts are a common practice in business. By offering such discounts, sellers encourage buyers to order in amounts greater than they would ordinarily purchase, thereby shifting the inventory holding cost from the seller to the buyer. Quantity discounts also reflect the savings inherent in large orders. For example, when the Citron Company receives an order from AAC, a certain amount of work is required to process the order, independent of its size. By selling a larger quantity of juicers, Citron is able to amortize this fixed cost over a larger quantity and charge a lower price for the merchandise.

Perhaps the most important reason firms offer quantity discounts is federal legislation known as the Robinson-Patman Act, which states that a seller cannot...
discriminate among buyers when setting prices. This means that Citron cannot charge one price to AAC (which sells 6240 juicers a year) and a different price to a nationwide discount chain selling, say, 624,000 juicers a year. A quantity discount schedule does, however, enable a seller to reward its biggest customers with lower prices without violating the Robinson-Patman Act.

**QUANTITY DISCOUNT SCHEDULES**

A quantity discount schedule lists the discounted cost per unit, $C_i$, corresponding to different purchase volumes. The quantities at which these prices change are called breakpoints, $B_i$. Each breakpoint corresponds to a particular pricing level. Normally, the price customers pay for the item declines as the order quantity increases.

Quantity discount schedules fall into two broad categories: all units schedules and incremental schedules. In an all units schedule, the price the buyer pays for all the units purchased is based on the total purchase volume. For example, suppose Citron uses an all units discount schedule and offers a discount price of $9.75 corresponding to a breakpoint of 300 juicers. If AAC orders 327 juicers, it will pay $9.75 per unit for each of the 327 juicers.

In an incremental discount schedule, the price discount applies only to the additional units ordered beyond each breakpoint. Thus if Citron uses an incremental discount schedule and offers a discount price of $9.75 corresponding to a breakpoint of 300 juicers, then if AAC orders 327 juicers, it will pay $10.00 per unit for the first 299 juicers and $9.75 per unit for the remaining 28 juicers. While some firms use incremental discount schedules, the all units discount schedule is more common.

**ALL UNITS DISCOUNT SCHEDULE**

Because the inventory unit cost is dependent on the quantity purchased, the all units discount inventory model must include the total cost of the goods purchased. The formula for $TC(Q)$, therefore, is as follows:

$$TC(Q) = \left( \frac{Q}{2} \right) C_0 + \left( \frac{D}{Q} \right) C_o + DC_i + C_o SS$$

(8.7)

where $C_i$ represents the unit cost at the $i^{th}$ pricing level corresponding to the order quantity, $Q$.

Since financing costs typically make up a major portion of the holding costs, it is reasonable to assume that the holding cost, $C_h$, will change proportionally to the unit cost. This assumption may not be totally accurate, however, because changes in the unit cost do not affect nonfinancing holding costs, such as storage costs. One way around this dilemma is to modify the holding cost by some fraction of the change in the unit cost reflecting the percentage of the holding cost represented by the inventory financing cost. Since the EOQ model is quite insensitive to minor changes in the parameters, however, the simplifying assumption that holding costs change proportionately to changes in the unit cost will have a minor effect on the optimal order quantity, $Q^*$.

**Determining the Optimal Order Quantity**

To illustrate how to determine the optimal order quantity under all units discount schedule, consider again the Allen Appliance Company example.

**ALLEN APPLIANCE COMPANY (CONTINUED)**

Mr. Allen was quite impressed with our analysis. After reading the memorandum, however, he realized that we had not been given complete information concerning the Citron juicers. In particular, while the base price for the Citron juicers is $10
per unit, Citron offers its customers quantity discounts. The all units quantity discount schedule is given in Table 8.5.

Mr. Allen wishes to determine whether he should order more than 300 units to take advantage of the discounts offered by Citron.

<table>
<thead>
<tr>
<th>Amount Ordered</th>
<th>Price per Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–299</td>
<td>$10.00</td>
</tr>
<tr>
<td>300–599</td>
<td>$9.75</td>
</tr>
<tr>
<td>600–999</td>
<td>$9.50</td>
</tr>
<tr>
<td>1000–4999</td>
<td>$9.40</td>
</tr>
<tr>
<td>5000 or more</td>
<td>$9.00</td>
</tr>
</tbody>
</table>

**SOLUTION**

There are four discount pricing levels for Citron juicers beyond the base price of $C = 10$ per unit. These are $C_1 = 9.75$, $B_1 = 300$; $C_2 = 9.50$, $B_2 = 600$; $C_3 = 9.40$, $B_3 = 1000$; and $C_4 = 9.00$, $B_4 = 5000$. Figure 8.8 shows five inventory cost curves for the juicer problem. Each represents the total cost $TC(Q)$ as a function of the corresponding unit price, $C_n$, assuming that $C_n$ is valid for all values of $Q$. The true total cost function, $TC(Q)$, contains the pieces of each curve corresponding to the range of values for $Q$ over which the corresponding discount price is valid, as highlighted in the figure.

![Figure 8.8 Inventory Cost Curves for AAC Juicer Problem—All Units Discount Policy](image)

Determining the optimal order quantity for an all units discount schedule is a straightforward process. For the original unit cost, as well as each discount pricing level, we use the EOQ formula

$$Q^* = \sqrt{\frac{2DC_0}{C_h}}$$
to determine the lowest total cost on each curve. Table 8.6 provides this information for the juicer problem. The \( Q^* \) values change slightly for each pricing level because \( C_h \) declines in proportion to the decrease in the price per unit for each pricing level. (If we had assumed a constant holding cost, \( Q^* \) would have been the same for each pricing level.)

### Table 8.6 Lowest Total Cost on Each Inventory Cost Curve

<table>
<thead>
<tr>
<th>Level</th>
<th>Amount Ordered</th>
<th>Price per Unit</th>
<th>( Q^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1–299</td>
<td>$10.00</td>
<td>327</td>
</tr>
<tr>
<td>1</td>
<td>300–599</td>
<td>$9.75</td>
<td>331</td>
</tr>
<tr>
<td>2</td>
<td>600–999</td>
<td>$9.50</td>
<td>336</td>
</tr>
<tr>
<td>3</td>
<td>1000–4999</td>
<td>$9.40</td>
<td>337</td>
</tr>
<tr>
<td>4</td>
<td>5000 or more</td>
<td>$9.00</td>
<td>345</td>
</tr>
</tbody>
</table>

It can be seen from Figure 8.8 that, for each pricing level, the cost curves increase as the order quantity increases beyond the \( Q^* \) value. Hence, when the EOQ value for a curve is below the pricing level’s breakpoint, the least expensive way to obtain the discount is to increase the order quantity only up to that breakpoint.

When the \( Q^* \) value is greater than the next pricing level’s breakpoint, as it is for pricing level 0, the pricing level can be dropped from further consideration (since we would be ordering at the next pricing level anyway). Obviously it does not make sense for Allen Appliance to order at pricing level 0 since its optimal order quantity, 327, falls within pricing level 1. But at level 1 the unit cost is $9.75, and the optimal order quantity is 331.

Table 8.7 lists the results of modifying the \( Q^* \) values to take advantage of the discounts for the remaining pricing levels under consideration. For each pricing level still under consideration the value of \( TC(Q^*) \) is then calculated by Equation 8.7 using the modified \( Q^* \) values. Table 8.8 gives the values of \( TC(Q^*) \) for each remaining pricing level (it is assumed \( SS = 13 \)).

### Table 8.7 Modified \( Q^* \) Values

<table>
<thead>
<tr>
<th>Amount Ordered</th>
<th>Price per Unit</th>
<th>Modified ( Q^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–299</td>
<td>$10.00</td>
<td>*</td>
</tr>
<tr>
<td>300–599</td>
<td>$9.75</td>
<td>331</td>
</tr>
<tr>
<td>600–999</td>
<td>$9.50</td>
<td>600</td>
</tr>
<tr>
<td>1000–4999</td>
<td>$9.40</td>
<td>1000</td>
</tr>
<tr>
<td>5000 or more</td>
<td>$9.00</td>
<td>5000</td>
</tr>
</tbody>
</table>

*\( Q^* \) above breakpoint.

### Table 8.8 \( TC(Q^*) \) Values

<table>
<thead>
<tr>
<th>Amount Ordered</th>
<th>Price per Unit</th>
<th>Modified ( Q^* )</th>
<th>( TC(Q^*) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–299</td>
<td>$10.00</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>300–599</td>
<td>$9.75</td>
<td>331</td>
<td>$61,309.88</td>
</tr>
<tr>
<td>600–999</td>
<td>$9.50</td>
<td>600</td>
<td>$59,821.09</td>
</tr>
<tr>
<td>1000–4999</td>
<td>$9.40</td>
<td>1000</td>
<td>$59,405.99</td>
</tr>
<tr>
<td>5000 or more</td>
<td>$9.00</td>
<td>5000</td>
<td>$59,341.36</td>
</tr>
</tbody>
</table>
Thus AAC should order 5000 juicers at a time, since this value minimizes \( TC(Q^*) \). The process is summarized as follows:

### Determining the Optimal Order Quantity for an All Units Pricing Schedule

1. Calculate \( Q^* \) for each discount level.
2. If \( Q^* \) is less than the lower quantity limit for the discount level, increase \( Q^* \) to this lower limit. If \( Q^* \) is greater than the upper quantity limit for the discount level, eliminate this level from further consideration.
3. Substitute the modified \( Q^* \) values into the formula:
   \[
   TC(Q) = \frac{Q}{2} C_h + \frac{D}{Q} C_s + DC + C_o SS
   \]
4. Select the \( Q^* \) value that minimizes \( TC(Q^*) \).

Although quantity discounts may influence the order quantity, they do not affect the reorder point; this is determined in exactly the same way as for the nondiscounted EOQ problem.

### Software Results

Figure 8.9 shows the results of using the All Units Discount worksheet on the inventory.xls template to determine the optimal order quantity and reorder point for the Allen Appliance Company.

![Excel Spreadsheet for Allen Appliance Company Problem with Discounts](Allen inventory.xls)

The parameters are entered in rows 5 through 11 of column B. (Note that the default value for \( Ch \) in cell B8 is determined by multiplying the value in cell B6, the cost per unit, by the value in cell B7, the annual holding cost rate.) The template can handle problems with up to eight breakpoints. The breakpoints
and discount prices are entered in rows 17 through 24 of columns B and C. (No entry is made for row 16 as the default value for cell C16 is the unit cost entered in cell B6.)

**Other Considerations**

These results are based on a mathematical model that made certain simplifying assumptions, such as constant demand and a fixed holding cost rate. However, the results should be checked to make sure they are consistent with these assumptions. For example, according to the above analysis, the lowest total annual inventory cost is achieved when AAC orders 5000 juicers at a time. This quantity represents more than a nine-month supply of juicers and could raise some concerns among AAC management.

First, if AAC orders 5000 juicers, it must have some place to store them. This could result in major additional expenses for new warehouse space. These additional costs would violate the fixed holding cost rate assumption.

Second, suppose Mr. Allen has heard rumors that Citron may be introducing a new improved juicer within the next three to four months. This plan could violate the constant demand assumption. AAC would rightfully be concerned about getting stuck with juicers that have been discontinued and need to be sold at drastically reduced prices.

Suppose, in fact, that AAC’s policy is to never order more than a three-month supply of any product in order to guard against such possibilities. Consequently, it would never order more than 6240/4 = 1560 juicers at a time. In this case, only the first three discount levels would be available, and, as seen in Figure 8.9, AAC should order 1000 juicers at a time.

**INCREMENTAL DISCOUNT SCHEDULES**

Figure 8.10 shows a graph of AAC’s total annual juicer inventory cost as a function of the order quantity, assuming that Citron offers an incremental discount schedule. The procedure for determining the optimal order quantity for an incremental discount schedule is somewhat more complex than that used for the all units schedule. Details of this procedure are given in Appendix 8.4 on the accompanying CD-ROM. The worksheet **Incremental Discount** in the Excel template **inventory.xls** can be used to obtain results for the discounted EOQ model if the discounting scheme is an incremental one.

![Figure 8.10](https://example.com/figure810.png)

*FIGURE 8.10* Total Annual Juicer Inventory Cost as a Function of Order Quantity—Incremental Discount Policy
8.5 Production Lot Size Model

Although the EOQ model is useful for determining an optimal inventory policy for goods obtained from other sources, in many instances the firm itself produces the items it sells. If demand for the item occurs at a constant rate, a production lot size model can be used to determine the item’s optimal inventory policy.

A production lot size model is useful for manufacturers such as pharmaceutical companies, soft drink bottlers, cosmetics companies, ice cream manufacturers, furniture makers, and household goods producers. In all of these enterprises, a production line is not continuously used to manufacture the same product; rather, production of an item occurs in batches, or lots, that are added to the firm’s inventory. Production does not resume until the item’s inventory is nearly depleted, at which point another batch is produced. An ice cream producer, for example, does not continuously produce Heavenly Hash ice cream. It makes a batch of this flavor, cleans out its equipment, and moves on to produce another flavor.

Production lot size models assume that the production facility operates at a rate greater than the demand rate for the item. Clearly, if the production rate is less than the demand rate, the firm will not have any inventory problem since it will simply ship out all items as they are produced. This situation sometimes occurs with the introduction of an extremely popular children’s toy or musical recording. Normally, this degree of popularity does not last long enough to warrant the firm expanding its production lines. Once demand declines below the production rate, a production lot size model may prove useful.

The discussion here focuses on determining the optimal production quantity for a single product. In the real world, where several products must share the same production line, it may be impossible to follow the resulting schedule for each product. Multiple scheduling problems are quite difficult to solve and are beyond the scope of this text.

The approach parallels that of the EOQ model by developing a general expression for the annual inventory costs associated with producing and storing the stock-keeping unit. In a production process, however, a firm does not actually place an order; instead, it incurs a cost to begin production, known as the setup cost. We use the following notation in production lot size models:

\[
\begin{align*}
D & = \text{Estimate of the Annual Demand for the Stock-Keeping Unit} \\
C_h & = \text{Annual Holding Cost per Unit in Inventory} \\
C_o & = \text{Production Setup Cost} \\
P & = \text{Annual Production Rate Assuming Full and Continuous Operation}
\end{align*}
\]

The goal of the optimal policy is to minimize total annual inventory costs. Because the production lot size model assumes that demand occurs at a constant rate, as with the EOQ model, we can show that, over an infinite time horizon, a stationary policy that produces the same quantity during each production run is optimal. Given a production lot size of \( Q \), an equation for the total annual variable cost, \( TV(Q) \), can be developed to represent the sum of the total annual holding cost plus production setup cost. \( Q^* \) denotes the value of \( Q \), which minimizes \( TV(Q) \).

**DETERMINING THE AVERAGE INVENTORY LEVEL**

The total annual holding cost is the product of the average inventory level and the annual holding cost per unit. To determine the average inventory level, the maximum inventory position, \( M \), must first be calculated. Unlike the EOQ model, in a production lot size model, the maximum inventory position is less
than the production lot size, Q. This is because units are demanded and sold while they are being produced. To determine the value of M, refer to Figure 8.11, which profiles the inventory position over time. Here, it is assumed that the production process operates at a constant rate and that production resumes only when the inventory is depleted.

FIGURE 8.11  Production Lot Size Model—Inventory Position over Time

The production cycle time, $T$, consists of two time periods: (1) the time during which the product is being produced and inventory is increasing, $T_1$; and (2) the time during which the production line is being used for other purposes and the good is not being produced, $T_2$.

Since production of $Q$ units takes place in time period $T_1$, then (assuming $T$ and $T_1$ are expressed on a yearly basis),

$$Q = PT_1$$

or

$$T_1 = Q/P$$

As Figure 8.11 indicates, the inventory position reaches its highest point during a production cycle at time $T_1$. At this point, production ceases and the inventory accumulated during the production phase of the cycle begins to be depleted. Because $Q$ units have been produced and $DT_1$ units have been demanded during time $T_1$, the inventory position at this point is:

$$M = Q - DT_1 = Q - D(Q/P) = Q(1 - D/P)$$

Total Annual Holding Costs

The average inventory level is, therefore:

$$M/2 = \left(\frac{Q}{2}\right) (1 - D/P)$$

and annual inventory holding cost is:

$$\left(\frac{Q}{2}\right) (1 - D/P)C_h$$
**Total Annual Setup Costs**

The annual production setup cost is found by multiplying the average number of production setups per year, \( \frac{D}{Q} \), by the setup cost incurred for each production run, \( C_o \):

\[
\left( \frac{D}{Q} \right) C_o
\]

**Total Annual Variable Costs**

Adding the annual holding cost to the production setup cost gives the following formula for \( TV(Q) \):

\[
TV(Q) = \frac{Q}{2} \left( 1 - \frac{D}{P} \right) C_h + \left( \frac{D}{Q} \right) C_o
\]

(8.8)

In Appendix 8.2 on the accompanying CD-ROM, it is shown that the optimal production quantity is given by the formula:

\[
Q^* = \sqrt{\frac{2DC_o}{(1 - \frac{D}{P})C_h}}
\]

(8.9)

As in the EOQ model, at \( Q^* \) the annual holding costs equal the annual production setup costs.

Comparing Equations 8.2 and 8.3 for the EOQ model to Equations 8.8 and 8.9 for the production lot size model, it can be seen that, if \( P \) equals infinity, \( \frac{D}{P} \) equals zero and the two sets of equations are identical. Hence the EOQ model can be considered a special case of the production lot size model, in which goods can be supplied infinitely quickly.

To illustrate use of the production lot size model, consider the problem faced by the Farah Cosmetics Company.

**FARAH COSMETICS COMPANY**

Management of the Farah Cosmetics Company is interested in determining the optimal production lot size for its most popular shade of lipstick, Autumn Moon. The factory operates seven days a week, 24 hours a day. The lipstick production line can produce 1000 tubes of lipstick per hour when operating at full capacity. Whenever the company changes production to a new shade of lipstick, it costs an estimated $150 to clean out the machinery and do any necessary calibrations.

Demand for Autumn Moon lipstick has been reasonably constant, averaging 980 dozen tubes per week. Farah sells the lipstick to distributors for $.80 per tube, and the firm calculates its variable production cost at approximately $.50 per tube. Because lipsticks must be stored in an air-conditioned warehouse, the firm uses a relatively high annual holding cost rate of 40%.

Farah is currently producing Autumn Moon in batch sizes of 84,000 tubes and would like to determine if this policy is optimal.
SOLUTION

Weekly demand of 980 dozen units is equivalent to a daily demand of 140 dozen, or 1680, units. Thus \( D = (1680)(365) = 613,200 \) per year. The annual holding cost, \( C_h \), for a tube of Autumn Moon lipstick is \((.40)(.50) = .20\), and the production setup cost, \( C_o \), is \( $150 \). For Autumn Moon, the maximum production rate, \( P \), is \((1000)(24)(365) = 8,760,000 \) units. The unit selling price of the lipstick, \$0.80, does not enter into the analysis, since revenue is unaffected by the production lot size decision. In summary:

\[
\begin{align*}
D &= 613,200 \\
P &= 8,760,000 \\
C &= .50 \\
C_o &= $150 \\
C_h &= .20
\end{align*}
\]

Analysis of Current Policy

Farah currently schedules production runs of 84,000 tubes of Autumn Moon lipstick so that the time between production runs, \( T \), is:

\[
T = \frac{84,000 \text{ tubes/run}}{613,200 \text{ tubes/year}} = .1370 \text{ years} \approx 50 \text{ days}
\]

Since the production line can manufacture 8,760,000 tubes per year, the length of each production run is:

\[
T_1 = \frac{84,000 \text{ tubes/run}}{8,760,000 \text{ tubes/year}} = .0096 \text{ years} \approx 3.5 \text{ days}
\]

Thus the time during each production cycle for which the machine is not used to produce Autumn Moon lipstick is:

\[
T_2 = T - T_1 = .1370 - .0096 = .1274 \text{ years} \approx 46.5 \text{ days}
\]

The total annual variable inventory cost for this policy is:

\[
TV(84,000) = \left( \frac{84,000}{2} \right) \left( 1 - \frac{613,200}{8,760,000} \right) (.20) + \left( \frac{613,200}{84,000} \right) ($150) = $8,907
\]

Analysis of Optimal Policy

Using Equation 8.9 the optimal production lot size is:

\[
Q^* = \sqrt{\frac{2(613,200)(150)}{1 - \frac{613,200}{8,760,000} (.20)}} \approx 31,449
\]

This quantity results in a total annual variable inventory cost of

\[
TV(31,449) = \left( \frac{31,449}{2} \right) \left( 1 - \frac{613,200}{8,760,000} \right) (.20) + \left( \frac{613,200}{31,449} \right) ($150) = $5,850
\]

This represents a variable inventory cost savings of $3057, or approximately 34%, over the policy of producing in batches of 84,000.
From a practical standpoint, the firm probably would not wish to schedule a production run of exactly 31,449 tubes of lipstick. A more realistic approach is to round off $Q^*$ and recommend a production quantity of 31,000 or 32,000, or even 30,000, tubes of lipstick. As with the EOQ model, small variations in $Q^*$ have a minimal effect on the total annual inventory costs.

**Software Results**

Figure 8.12 shows the results of using the worksheet **Production Lot Size** on the *inventory.xls* template for determining the inventory policy for the Farah Cosmetics Company. The parameters are entered in column B, and the optimal policy outputs are given in column E. One can also assign an order quantity in cell H5 (in this case $Q = 84,000$), and the resulting inventory values will be given in cells H6 through H10.

As with the EOQ model, one can use the Solver option to also determine the optimal production lot size. To do this, click on Solver and in the “Set Target Cell” box put $\$H9$, in the “Equal To” section highlight the Min button, and in the “By Changing Cells” box put $\$H5$. Next click Solve. The EOQ solution will appear in cell H5 and the values in column H will be identical to those in column E.

**OTHER PRODUCTION SYSTEMS**

Many other inventory systems exist for controlling the production of inventory. Supplement CD 6 on the accompanying CD-ROM discusses a number of these systems, including material requirements planning (MRP), just-in-time systems, kanban systems, and flexible manufacturing systems. Also discussed in this supplement are the Wagner-Whitin algorithm and the Silver-Meal heuristic.
8.6 Planned Shortage Model

BACKORDERING

When we go to our local supermarket, we expect to find items like eggs, coffee, spaghetti, and ketchup readily available. If they aren’t, we will probably begin shopping elsewhere. In Section 8.3 we discussed a safety stock approach that businesses can use to reduce the likelihood of running out of stock of such items. In other situations, however, we have become accustomed to waiting several days or even weeks or months to get the merchandise we want.

For example, if we wish to purchase a new car complete with a detailed list of specific options, and the car is not currently available at our local new car dealership, it may take the factory six or more weeks to produce and deliver the car. Other products we are typically willing to wait for include quality furniture, some major appliances, and specialty parts. What these items have in common are relatively high holding costs, due to either the high cost of the item or its low demand.

The phenomenon of waiting for merchandise to be delivered is known as backordering. If an item desired by a customer is not available, the customer either goes elsewhere for the good (a lost sale) or places a backorder for the item. When the next shipment of the item arrives, all backorders are filled immediately and the remainder of the order is placed into inventory. One approach used to represent such situations is the planned shortage model.

MODEL ASSUMPTIONS

In a planned shortage model, it is assumed that no customers will be lost due to an out-of-stock situation; all such customers will backorder. Although this assumption may not necessarily be true if the item or an appropriate substitute is readily available from another source, it might be appropriate if there is no alternative to waiting. Like the EOQ and production lot size models, this model deals with an item with a sufficiently long shelf life, whose demand occurs at a known constant rate over an infinite time horizon. In addition to holding and ordering costs, this model allows us to incorporate both a time-dependent and a time-independent shortage cost component.

Naturally, customers prefer not to have to wait for their merchandise. To incorporate this preference, the planned shortage model includes a cost, \( C_s \), of keeping a customer on backorder for an entire year. (The cost for waiting less than a year is simply prorated.) For example, if a store offers a $10 per week discount for each week, a customer waits for merchandise, then \( C_s = $10(52) = $520 \). Hence, if a customer has to wait only four weeks for the item, the backorder cost for that customer is \( (4/52)(520) = $40 \).

In general, \( C_s \) does not represent a customer discount, but, rather, an estimate of customer dissatisfaction known as a goodwill cost. This cost translates into the future reduction in the firm’s profitability associated with keeping a customer waiting. Goodwill costs are, at best, difficult to measure, but firms can sometimes get reasonable estimates of such costs from marketing surveys and focus groups.

In addition to the goodwill cost of keeping a customer waiting for the item, an administrative cost, \( C_{ab} \), may also be associated with writing up a backorder and contacting the customer whose item was on backorder when the order arrives. This cost differs from \( C_s \) because it is a fixed cost per backorder, independent of how long a customer waits for the item to arrive.

In the planned shortage model, the decision maker can control two quantities: \( Q \), the order quantity; and \( S \), the number of units on backorder at the time the
next order arrives. Hence, the total variable inventory cost equation, \(TV(Q, S)\), is a function of both of these quantities:

\[
TV(Q, S) = (\text{Total Annual Holding Costs}) + (\text{Total Annual Ordering Costs}) + (\text{Total Annual Time-Dependent Shortage Costs}) + (\text{Total Annual Time-Independent Shortage Costs})
\]

Figure 8.13 is useful in developing expressions for these terms. In this figure:

- \(T_1\) = the period in the inventory cycle during which inventory is available
- \(T_2\) = the period in the inventory cycle during which items are on backorder
- \(T\) = the time of the complete inventory cycle = \(T_1 + T_2\)

An inventory cycle begins when \(S\) units are on backorder and an order of size \(Q\) is received. Since the \(S\) backorders are filled immediately, the inventory position at the beginning of the cycle is brought to its maximum inventory position \(M = Q - S\). The model assumes that items are demanded (thus, inventory is depleted) at a constant rate until the inventory level reaches 0 at time \(T_1\). Then, during period \(T_2\), backorders accumulate at the same constant demand rate until, at the end of the cycle, \(S\) units are on backorder. At that point the next cycle begins, and the process is repeated. The individual components of \(TV(Q, S)\) can now be determined.

**ANNUAL HOLDING COST**

The average inventory level during the entire cycle, \(T_1\), is the average inventory level during the in-stock period, \((Q - S)/2\), times the proportion of time that there is inventory in stock \((T_1/T)\). The demand during period \(T_1\) is \(D \times T_1 = Q - S\), while the demand during the entire cycle \(T\) is \(D \times T = Q\). Dividing \(Q - S\) by \(Q\) therefore gives the value of \(T_1/T\). Hence we have that:

\[
\text{Average Inventory Level} = \left(\frac{(Q - S)}{2}\right) \times \left(\frac{(Q - S)}{Q}\right) = \frac{(Q - S)^2}{2Q}
\]

As in the previous models, the annual holding cost is the average inventory level times \(C_h\). Thus

\[
\text{Annual Inventory Holding Costs} = \frac{(Q - S)^2}{2Q} C_h
\]
ANNUAL ORDERING COST
The annual ordering cost is found by multiplying the average number of orders per year, \( N = \frac{D}{Q} \), by the fixed ordering cost, \( C_o \). Thus the total annual ordering cost is:

\[
\left( \frac{D}{Q} \right) C_o
\]

ANNUAL TIME-DEPENDENT SHORTAGE COST
The average backorder level during the cycle is the average number of stockouts during the out-of-stock period, \( S/2 \), times the proportion of time inventory is on backorder, \( \frac{T_2}{T} \). Since \( T_1 = \frac{Q - S}{Q} \), and \( T_1 + T_2 = 1 \), it follows that \( T_2 = \frac{S}{Q} \)

\[
\text{Average Backorder Level} = \left( \frac{S}{2} \right) \left( \frac{S}{Q} \right) = \frac{S^2}{2Q}
\]

The annual time-dependent shortage cost equals the average backorder level times \( C_b \). Thus

\[
\text{Annual Time-Dependent Backorder Cost} = \left( \frac{S^2}{2Q} \right) C_b
\]

ANNUAL TIME-INDEPENDENT SHORTAGE COST
During each inventory cycle there are \( S \) backorders, and there are \( \frac{D}{Q} \) cycles per year, so that

\[
\text{Total Number of Backorders During a Year} = \left( \frac{D}{Q} \right) S
\]

The annual shortage cost that is incurred independent of the length of time customers are on backorder is \( C_b \) times the total number of backorders during the year. Thus

\[
\text{Annual Time-Independent Backorder Cost} = \left( \frac{D}{Q} \right) SC_b
\]

TOTAL ANNUAL COSTS
Combining these terms gives the following formula for \( TV(Q, S) \):

\[
TV(Q, S) = \frac{(Q - S)^2}{2Q} C_h + \frac{D}{Q} (C_o + SC_b) + \frac{S^2}{2Q} C_s
\]  

(8.10)

OPTIMAL INVENTORY POLICY
Let \( Q^* \) and \( S^* \) represent the pair of values for \( Q \) and \( S \) that minimize \( TV(Q, S) \). Although it is not always possible to obtain closed-form solutions for a pair of values such as \( Q^* \) and \( S^* \), they can be obtained for this model provided that \( C_i > 0 \) and

\[
C_b < \sqrt{2C_o C_i / D}
\]
Optimal Inventory Policy for the Planned Shortage Model

\[ Q^* = \sqrt{\frac{2DC_o}{C_h}} \left( \frac{C_b + C_s}{C_s} \right) - \frac{(DC_b)^2}{C_h C_s} \]  
(8.11)

and

\[ S^* = \frac{Q^*C_h - DC_b}{C_h + C_s} \]  
(8.12)

REORDER POINT

To find the reorder point when there is a lead time of \( L \) years, we note that \( S^* \) represents the number of orders on backorder when the new shipment arrives. If the annual demand is for \( D \) units, the lead time demand equals \( L \cdot D \). Hence the reorder point, \( R \), for this model is given by the following formula:

\[ R = L \cdot D - S^* \]  
(8.13)

The value of \( R \) can be negative, implying that the reorder point occurs when several units are already on backorder. If \( R = -5 \), for example, the order should be placed when five units are on backorder.

To illustrate the planned shortage model, consider the situation faced by the Scanlon Plumbing Corporation.

SCANLON PLUMBING CORPORATION

Scanlon Plumbing Corporation is the exclusive North American distributor of a portable sauna manufactured in Sweden. The saunas cost Scanlon $2400 each, and the company estimates the annual holding cost per unit for this product is $525. Because the saunas must be shipped in a containerized vessel, the fixed ordering cost is fairly high, at $1250. Lead time for delivery is four weeks.

Scanlon receives orders for an average of 15 saunas per week. Customers are willing to place orders for the saunas when Scanlon is out of stock. However, the company estimates the goodwill cost of keeping a customer’s sauna on backorder is $20 per week. There is also an administrative cost of $10 for each sauna placed on backorder.

Management wishes to determine an optimal inventory policy for ordering the saunas.

SOLUTION

For the Scanlon Plumbing model the parameters are:

\[ D = 15(52) = 780 \]
\[ C_o = $1250 \]
\[ C_h = $525 \]
\[ C_s = $20(52) = $1040 \]
\[ C_b = $10 \]
Substituting these quantities into Equations 8.11 and 8.12 gives:

\[ Q^* = \sqrt{\frac{(2)(780)(1250)}{525}} \frac{525 + 1040}{1040} - \frac{(780)(10)^2}{(525)(1040)} \approx 74 \]

and

\[ S^* = \frac{(74)(525) - (780)(10)}{525 + 1040} \approx 20 \]

Since lead time is four weeks, \( L = 4/52 = .07692 \) years. Orders should therefore be placed when the inventory level reaches:

\[ R = .07692(780) - 20 = 40 \text{ units} \]

**PRACTICAL CONSIDERATIONS**

Again, the validity of the assumptions we used to generate \( Q^* \) and \( S^* \) should be evaluated. For example, the planned shortage model assumes that goodwill costs are linear; that is, the cost of keeping a customer on backorder for four months is assumed to be four times the cost of keeping a customer on backorder for one month. This is probably not the case in most real-world situations. Customers may be reasonably tolerant of short delays, but long delays can result in a tremendous loss in customer goodwill. Hence the decision maker should analyze the model results to see whether the assumed backorder cost is realistic.

In this case, it was assumed that the backorder cost for saunas is $20 per week. Although management feels that this cost is realistic for delays of a week or less, it believes that delays greater than a week are actually more costly. The maximum delay experienced by any customer can be found by dividing \( S^* \) by \( D \). For Scanlon Plumbing, the maximum backorder delay encountered by a customer (in weeks) is \( S^*/D = 20/15 = 1\frac{1}{3} \) weeks.

As a result of this analysis, management might feel that a $25 per week backorder cost is more realistic. This variation changes \( C_s \) from $1040 to $1300 and, using Equations 8.11 and 8.12, suggests a revised inventory policy of ordering 72 units when the inventory level reaches 44 units. In this case, the maximum number of customers on backorder is approximately 16, and the longest time a customer will spend on backorder is just slightly more than one week.

**ECONOMIC IMPLICATIONS**

It is worth examining the implications of the formulas for \( Q^* \) and \( S^* \) on inventory control. (To simplify, assume \( C_h \) equals 0.) If \( C_h \) is quite large relative to \( C_s \) (as it is for custom or big-ticket items for which the buyer expects delays), the ratio \( C_h/(C_h + C_s) \) is close to 1 and \( S^* \) is approximately \( Q^* \). That is, whenever an inventory order arrives, nearly all units have already been presold and the firm carries virtually no inventory. The optimal order quantity in this case is given approximately by the formula

\[ Q^* = \sqrt{\frac{2DC_h}{C_s}} \]

In this case, the holding cost plays no role in determining \( Q^* \).

On the other hand, now suppose that \( C_s \) is quite large relative to \( C_h \), as it is for products for which customer goodwill costs are quite high if the item is not available for purchase (such as daily staples like bread, milk, and eggs). In this case, \( C_s/(C_h + C_s) \) is close to 0, and the firm almost always wants the good to be in
stock. Therefore, customers are never intentionally placed on backorder. Furthermore, because \((C_h + C_b)/C_s\) is effectively 1, the formula for \(Q^*\) reduces to the EOQ formula (Equation 8.3). Hence we can view the EOQ model as a special case of the planned shortage model for which \(C_s\) equals infinity and \(C_b\) equals 0.

Thus, if an item has a high holding cost relative to its backorder cost, the firm will carry little, if any, inventory, while if its backorder cost is high relative to its holding cost, the firm generally tries to keep the item in inventory and avoid running out of stock.

### SPECIAL CASES

A special case of the planned shortage model occurs when the value of \(C_b\) is high relative to the value of \(C_s\). In particular, when \(C_b > \sqrt{2C_s/C_h}\), the optimal solution is to allow no shortages \((S^* = 0)\), and \(Q^*\) equals the EOQ value.

To understand why this is the case, note that, for the EOQ solution, \(Q^* = \sqrt{2DC_s/C_h}\). Substituting this value into the total annual variable cost formula (Equation 8.1) gives:

\[
TV(Q^*) = \sqrt{2DC_sC_h} \tag{8.14}
\]

Dividing \(TV(Q^*)\) by the annual demand, \(D\), results in the variable inventory cost per unit under the EOQ policy, \(\sqrt{2C_s/C_h}\). Hence, if the administrative backorder cost, \(C_b\), is greater than this amount, it must be more expensive to allow backorders than not to allow them.

Another special case of the model occurs when \(C_s = 0\). If the value of \(C_s\) is 0 and \(C_b \leq \sqrt{2C_s/C_h}/D\), the model does not make any sense because the optimal policy is to keep all customers on backorder for an infinitely long period of time.

### Software Results

Figure 8.14 gives results of using the Planned Shortage worksheet on the inventory.xls template to determine the optimal order quantity and reorder point for the Scanlon Plumbing Corporation problem.

**FIGURE 8.14** Excel Spreadsheet for Scanlon Plumbing Corporation
In this spreadsheet the parameters are entered in column B. Note that there are entries for $C_s$, the cost of keeping a unit on backorder for one year, and $C_b$, the fixed administrative cost of putting a unit on backorder.

The values in cells E5 and E6 give the optimal values for the order quantity, $Q^*$, and the number of units on backorder when the order arrives, $S^*$. Cells H5 and H6 allow the user to input specified values for $Q$ and $S$ (in this case 74 for $Q$ and 20 for $S$).

As with the EOQ model, one can use the Solver option to also determine the optimal values for $Q$ and $S$. To do this, click on Solver and in the “Set Target Cell” box put $H10$, in the “Equal To” section, highlight the Min button, and in the “By Changing Cells” box enter $H5$-$H6$. For this model, however, we must make sure that $Q$ and $S$ are $> 0$. Since these are strict inequalities, adding $Q \geq 0.00001$ and $S \geq .00001$ in the “Add Constraints” dialogue box roughly approximates these constraints. When Solve is clicked the optimal values for $Q$ and $S$ will appear in cells H5 and H6 respectively and the values in column H will be identical to those in column E.

### 8.7 Review Systems

#### CONTINUOUS REVIEW SYSTEMS

The EOQ model, the production lot size model, and the planned shortage model are all examples of continuous review systems because we implicitly assume that the inventory position is *continuously* monitored and an order is placed at the instant the inventory level reaches the reorder point.

**$(R,Q)$ Policies**

For the EOQ, production lot size, and planned shortage models, an order of size $Q$ is placed whenever the inventory level reaches the reorder point, $R$. Hence these models are sometimes known as *order point, order quantity, or $(R,Q)$ policies*.

There are several ways to implement such policies. If inventory is tracked by a point-of-sale (POS) computerized cash register system, as it is in many retail establishments such as department stores and supermarkets, the computer can be programmed to carry out the recommended policy. Because the computer does not record shrinkage (theft and breakage), however, this method is not foolproof. This shortcoming is generally not too severe, however, and can be overcome by introducing a shrinkage factor into the computer program.

If the inventory is not tracked by computer, it may appear impossible, from a practical standpoint, to continuously review the firm’s inventory position. Fortunately, there is a fairly easy way of implementing an $(R,Q)$ policy, known as the *two-bin system*. This system, which is used in a number of factories and supply houses, operates as follows.

For a given product, a small bin holds $R$ units and a larger bin holds up to $Q$ units. When a new order arrives, the small bin is filled and tagged, and the remaining inventory from the order is placed in the large bin. Employees are instructed to remove units from the large bin until that bin is depleted. Once the large bin is empty, the small bin is opened up and items are supplied from that bin. At the time the small bin is opened, the order tag is removed and given to the inventory foreman. The order tag alerts the foreman that it is time to place an order for an additional $Q$ units.

**$(R,M)$ Policies**

Implicit in the development of our previous models is the fact that the firm sells stock-keeping units one at a time. In many businesses, however, the typical cus-
tomer order may consist of multiple units of the same item. When such an order triggers the reorder point, problems can arise in an (R,Q) system.

For example, suppose that Citron did not offer quantity discounts and AAC adopted the policy recommendation to order \( Q = 327 \) units when the inventory level reaches \( R = 219 \). If the inventory level is 224 and one of AAC's customers purchases 60 juicers, this purchase triggers a new order. If AAC only orders 327 juicers, it may find itself out of stock again faster than anticipated because the inventory position when the order is placed, \( I \), will only be \( I = 224 - 60 = 164 \) juicers. This will be \( R - I = 219 - 164 = 55 \) units below the reorder point.

One way to avoid the potential problem caused by exceptionally large orders is to use an order point, order up to level, or (R, M) policy. Under this system, while a new order is placed whenever the inventory level falls to \( R \) or below, the order size is adjusted to \( Q + (R - I) \) to bring the inventory level back up to an anticipated level of \( M, (M = Q + SS) \). This system effectively amends the (R,Q) policy to account for situations in which the inventory level may be substantially below the reorder point when the order is placed.

### Adjusted Order Quantity for (R,M) Model

\[
\text{Order Quantity} = Q + (R - I) = (M - SS) + (R - I)
\]

For example, suppose AAC uses an (R, M) policy with reorder point \( R = 219 \), and anticipated maximum inventory level \( M = 354 \) (the order quantity of 327 plus the safety stock of 27). If a customer orders 60 juicers when the inventory level is at 224, a reorder is triggered when the inventory position, \( I \), equals \( 224 - 60 = 164 \). Hence the reorder amount should be equal to \( 354 - 27 + 219 - 164 = 382 \) instead of 327 juicers. The difference between 382 and 327, 55, reflects the fact that when the order is placed, the inventory level is 55 units below the desired reorder point of 219.

In the long run, an (R, M) policy typically has a lower average cost than a comparable (R, Q) policy. (R, M) systems are more complicated to monitor than (R, Q) systems, however. For example, implementing an (R, M) policy in a two-bin system requires the employee who removes the tag from the smaller bin to record how many units are being removed from that bin at the time it is opened. This extra control cost is not always worth the savings achieved by the (R, M) system.

### PERIODIC REVIEW SYSTEMS

Continuous review systems are not practical for many businesses. Some establishments may not have the resources available to purchase computerized cash register systems or the space available to adopt a two-bin system. Others may order many different items from the same vendor and find it impractical to place separate purchase orders at different time periods for the numerous SKUs. In such instances, many firms use a periodic review inventory system. In this system the inventory position for each SKU is observed at fixed periods at which time order decisions are made.

### (T, M) Policies

In replenishment cycle or (T, M) policies, the inventory position is reviewed every \( T \) time units (days, weeks, etc.), and an order is placed to bring the inventory level for the stock-keeping unit back up to an anticipated maximum inventory level, \( M \). This anticipated maximum level is determined by forecasting the number of units demanded during the review period and adding the desired safety stock to this amount. Replenishment cycle policies are typically used by rack jobbers who have
a scheduled plan for servicing customers. The following formulas can then be used to find the order quantity, \( Q \), and the value of \( M \), for a \((T, M)\) policy:

\[
M = T \cdot D + SS
\]

Optimal Order Quantity:
\[
Q = M + L \cdot D - I
\]

Where:
- \( T \) = Review Period
- \( L \) = Lead Time
- \( D \) = Demand
- \( SS \) = Safety Stock
- \( I \) = Inventory Position

To illustrate this technique, let us return to the situation at the Allen Appliance Company.

**ALLEN APPLIANCE COMPANY (CONTINUED)**

Allen Appliance Company has begun selling several different products from Citron in addition to its juicers and has decided to implement a periodic review system for controlling inventory. Citron makes regular deliveries to AAC every three weeks, based on orders it has received eight days before shipment. Thus AAC reviews its inventory every three weeks, eight days before it expects a shipment, and faxes an order to Citron. It is now time for AAC to place an order, and it finds that 210 juicers are in stock. AAC wants to know how many juicers to order if it now desires a safety stock of 30 units. Recall that Allen operates 260 days per year.

**SOLUTION**

For this model we will express \( T \), \( L \), and \( D \) in years. Hence \( T = 3 \) weeks or \( 3/52 \approx 0.05769 \) years, \( L = 8 \) days or \( 8/260 \approx 0.03077 \) years, \( D = 6240 \) units per year, \( SS = 30 \) units, and \( I = 210 \) units. Thus the anticipated maximum inventory, \( M \), is:

\[
M = 360 + 30 = 390 \text{ juicers}
\]

and the order quantity should be:

\[
Q = 390 + (0.03077)(6240) - 210 = 372 \text{ juicers}.
\]

**\((T, R, M)\) Policies**

One shortcoming of a \((T, M)\) policy arises when there has been little demand for the stock-keeping unit during the previous review period. Under these circumstances, it may not even pay to place a new order. We can modify the \((T, M)\) policy by introducing a threshold inventory level \( R \), meaning that orders for the stock-keeping unit are placed only if the current inventory level is \( R \) or less. This policy, known as a \((T, R, M)\) policy, has a lower long-run cost than a simple \((T, M)\) policy. How the optimal values for \( T \) and \( R \) are determined is beyond the scope of this text, however, this problem can be analyzed using the simulation approach detailed in Chapter 10.
8.8 Single-period Inventory Model

The inventory models developed thus far assumed that demand occurs at a known and reasonably constant rate and that the shelf life of the stock-keeping units is long enough that the goods will not spoil or deteriorate in value while in inventory. In many situations, however, demand is stochastic, and the inventory shelf life or selling season is quite short. Consequently, the stock-keeping unit cannot be carried in inventory beyond a certain time period. In such cases, we can use the single-period inventory model to determine the optimal order quantity. This model assumes that demand varies according to a specified probability distribution and deals with a stock-keeping unit that has a limited shelf life of one period. Hence no inventory is stored from period to period.

A common example is a newsstand where newspapers that are unsold at the end of a day are not kept in inventory but are disposed of to make room for delivery of the next day’s papers. For this reason, the single-period inventory model is frequently referred to as the newsboy problem. Other commodities that have a limited shelf life include magazines, Christmas trees, Valentine’s Day candy, Halloween costumes, baked goods, concert tickets, chemicals, seasonal clothing, and dairy products. The basic assumptions of the single-period inventory model are as follows.

Assumptions of the Single-period Inventory Model

1. Inventory is saleable only within a single time period.
2. During each time period, inventory is delivered only once.
3. Customer demand during each period is stochastic (random) but follows a known probability distribution.
4. At the end of each period, unsold inventory is disposed of for some salvage value (which may be positive, negative, or zero).
5. The salvage value is less than the cost of the good.
6. If customer demand exceeds available supply, the firm may encounter a goodwill or shortage cost for each unsatisfied customer demand.

To determine the optimal order quantity in a single-period inventory model, a balance must be struck between overordering, which would create excess inventory left at the end of the period, and underordering, which would result in unsatisfied customer demand. The following notation is used for single-period inventory models:

\[
\begin{align*}
    p &= \text{per unit selling price of the good} \\
    c &= \text{per unit cost of the good} \\
    s &= \text{per unit salvage value of unsold goods} \\
    g &= \text{goodwill or shortage cost for each unsatisfied customer} \\
    K &= \text{fixed purchasing costs} \\
    Q &= \text{order quantity} \\
    \text{EP}(Q) &= \text{expected profit if } Q \text{ units are ordered}
\end{align*}
\]

DEVELOPMENT OF THE SINGLE-PERIOD MODEL

Our analysis begins by developing an expression for \( \text{EP}(Q) \). Two scenarios are possible depending on whether the actual demand, \( x \), is (1) less than the order quantity, \( Q \); or (2) greater than or equal to the order quantity, \( Q \). Each leads to a different expression for the profit.
Case 1: Demand, $x$, Is Less Than the Number of Units Stocked, $Q$

In this case, $x$ units are sold at a price of $p$ each, and the remaining $(Q - x)$ units will have to be disposed of for a salvage value of $s$ each. Hence, total revenue is $px + s(Q - x)$. Since demand is less than the available inventory, no goodwill costs are incurred. The cost of stocking the $Q$ units is $cQ + K$. Accordingly, profit is given by the following expression:

$$\text{Profit} = px + s(Q - x) - cQ - K$$

Case 2: Demand, $x$, Is Greater Than or Equal to the Number of Units Stocked, $Q$

In this case, all $Q$ units are sold at a price of $p$ each, and no inventory is left to dispose of for salvage value. Hence the total revenue is $pQ$. As demand exceeds the available inventory, $(x - Q)$ customer demands are unsatisfied at a goodwill cost of $g each. The cost of supplying the $Q$ units is $cQ + K$, bringing total costs to $g(x - Q) + cQ + K$. Hence the profit is:

$$\text{Profit} = pQ - g(x - Q) - cQ - K$$

Note that, in either case, the profit varies according to the actual demand, $x$. The expected profit, $EP(Q)$, given an order quantity of $Q$, is determined by finding the weighted average profit over all possible values of $x$. When demand follows a discrete probability distribution, $p(x)$, this is done by multiplying the profit corresponding to a demand of $x$ by the probability that $x$ units are demanded and then summing these products over all possible values of $x$:

**Expected Profit for the Discrete Demand Distribution**

$$EP(Q) = \sum_x (\text{profit given demand} = x) \times p(x)$$

Substituting the formulas for the profit in the two demand cases yields a straightforward, but rather messy, formula for $EP(Q)$.

If demand is assumed to follow a continuous probability distribution with density function $f(x)$, the expected profit function can be determined as follows:

**Expected Profit for the Continuous Demand Distribution**

$$EP(Q) = \int (\text{profit given demand} = x) \times f(x)dx$$

The objective is to determine the value, $Q^*$, that maximizes this expected profit function. Appendix 8.6 on the accompanying CD-ROM contains detailed formulas for $EP(Q)$ as well as the derivation of $Q^*$ for both discrete and continuous probability demand distributions. There it is shown that when demand follows a discrete probability distribution with cumulative probability $P(x)$, $Q^*$ is the smallest value of $Q$, such that:

$$P(D \leq Q^*) \geq \frac{p - c + g}{p - s + g} \quad (8.15)$$
When demand follows a continuous probability distribution with cumulative distribution \( F(x) \), \( Q^* \) satisfies the following relationship:

\[
F(Q^*) = \frac{p - c + g}{p - s + g}
\]  

(8.16)

Note that the fixed cost, \( K \), plays no role in determining the optimal order quantity. The ratio \( (p - c + g)/(p - s + g) \) is known as the optimal service level. It represents the long-run percentage of periods in which inventory is sufficient to satisfy all customer demands. This does not imply that in any given period the proportion of customer demands that will be satisfied is \( (p - c + g)/(p - s + g) \).

**A SINGLE-PERIOD MODEL WITH A DISCRETE DEMAND DISTRIBUTION**

To help motivate the single-period inventory model when demand can be modeled by a discrete distribution, consider the situation faced by the Sentinel newspaper.

**THE SENTINEL NEWSPAPER**

Company management at the Sentinel newspaper wishes to determine how many papers should be placed in each of several vending machine locations. The newspapers sell for $0.30 each. Although they cost the company $0.38 each to produce, Sentinel receives approximately $0.18 in advertising revenue for each paper printed. Hence the net production cost per paper is $0.20.

Any unsold papers are delivered to a newspaper recycling firm and net the Sentinel $0.01 per copy. The company estimates that if a vending machine runs short of papers, it will suffer a goodwill loss of $0.10 for each unsatisfied customer. The cost of filling a vending machine averages $1.20.

Based on many prior weeks of sales data, the circulation manager for the Sentinel estimates that demand for the Tuesday paper at the Sixth and Main vending machine location approximately follows a discrete uniform distribution with demand between 30 and 49 papers. Management would like to know the optimal number of papers to stock at this location.

**SOLUTION**

In this model

- \( p = \) selling price per paper = $0.30
- \( c = \) cost per paper = $0.20
- \( s = \) salvage value per paper = $0.01
- \( g = \) goodwill cost per paper = $0.10
- \( K = \) fixed cost = $1.20

To illustrate the two profit expressions, suppose the Sentinel puts 40 newspapers in a vending machine. If demand is only 32 newspapers, this is Case 1 since 32 \(<\) 40. Hence, the profit would be $0.30(32) + $0.01(40 - 32) - $0.20(40) - $1.20 = $0.48.

On the other hand, if demand is 45 newspapers, this is Case 2 since 45 \(\geq\) 40. Now the profit would be $0.30(40) - $0.10(45 - 40) - $0.20(40) - $1.20 = $2.30.
The objective is to determine the optimal number of newspapers, \( Q^* \), to stock at the Sixth and Main location. Substituting the values of the parameters for the Sentinel newspaper into the right side of Equation 8.15 gives the following optimal service level:

\[
\frac{p + g - c}{p + g - s} = \frac{.30 + .10 - .20}{.30 + .10 - .01} = \frac{.20}{.29} = .513
\]

Hence \( Q^* \) is the smallest value for \( Q \) such that \( P(D \leq Q^*) \geq .513 \). Because demand is uniformly distributed between 30 and 49 papers, demand will equally likely be 30, 31, 32, \ldots, 48, or 49. For these 20 possible values, \( p(x = 30) = p(x = 31) = p(x = 32) = \ldots = p(x = 49) = 1/20 = .05 \); therefore, \( P(x \leq 30) = .05 \), \( P(x \leq 31) = .10 \), \( P(x \leq 32) = .15 \), and so on. Proceeding in this fashion, \( P(x \leq 39) = .50 \) and \( P(x \leq 40) = .55 \).

Since \( P(x \leq 39) = .50 \) is less than .513, and \( P(x \leq 40) = .55 \) is greater than .513, the Sentinel should put 40 newspapers in the vending machine. Note that, given the discrete nature of the probability distribution, stocking the vending machine with 40 papers actually results in a 55% service level.

**Software Results**

Figure 8.15 shows an Excel spreadsheet that can be used to determine the order quantity for a single-period inventory model in which demand follows a uniform distribution.

**FIGURE 8.15** Excel Spreadsheet for the Sentinel Newspaper

In this spreadsheet the parameters are entered in column B. The outputs for the problem, the optimal service level, the optimal order quantity, and the resulting service level are calculated in cells E5, E6, and E7, respectively.

The worksheet Single-Period (Uniform) on the inventory.xls template can be used to determine the optimal inventory policy for single-inventory models in which demand follows a uniform distribution. The format for the worksheet is similar to that used in Figure 8.15.

**A SINGLE-PERIOD MODEL WITH A CONTINUOUS DEMAND DISTRIBUTION**

To illustrate these concepts when there is a continuous probability distribution for demand, consider the case of Wendell’s Bakery.
WENDELL'S BAKERY

Vermont State University has given Wendell’s Bakery the concession to sell glazed donuts in the School of Business during evening classes. It costs the bakery $.15 to produce each donut, and Wendell’s sells the donuts to students for $.35 each. Any donuts that are unsold at the end of the evening are donated to a local charity, and the bakery obtains a tax credit equal to $.05 per donut. If Wendell’s produces too few donuts and does not have enough to satisfy all its customers, it incurs a customer goodwill cost of $.25 per donut demanded. The bakery hires a student to operate the concession and pays the student $15 per evening.

After several months of operations, the bakery has determined that demand for glazed donuts on weekday evenings approximately follows a normal distribution with a mean of 120 and a standard deviation of 20. Wendell’s wishes to determine the optimal number of donuts to prepare for weekday evenings.

SOLUTION

For this problem,

\[
p = \text{selling price per donut} = $\ .35 \\
c = \text{cost per donut} = $\ .15 \\
s = \text{salvage value per donut} = $\ .05 \\
g = \text{goodwill cost per donut} = $\ .25 \\
K = \text{fixed operating cost} = $15.00
\]

Hence \( Q^* \) should be chosen so that:

\[
F(Q^*) = \frac{(p - s) - c}{(p - s) - c + s - g} \approx .8182.
\]

That is, the bakery should produce enough donuts so that the probability of satisfying all demands during the evening is .8182.

To determine values for the normal distribution, the standard normal random variable, \( z \) is used. Recall that \( z \) represents the number of standard deviations that a variable, \( x \), is from its mean; \( z \) is related to \( x \) by the formula \( z = (x - \mu)/\sigma \), or \( x = \mu + z\sigma \).

In Figure 8.16 it is shown that a cumulative probability of .8182 corresponds to the shaded area of .3182 between 0 and the appropriate \( z \) value. Referring to Appendix A, the closest entry in the table to .3182 is .3186, corresponding to a \( z \) value of .91. Thus, using \( z = .91 \), the number of donuts the bakery should deliver to the School of Business is:

\[
Q^* = \mu + z\sigma = 120 + .91 \times 20 \approx 138
\]

If demand follows a normal distribution with a mean of \( \mu \) and a standard deviation of \( \sigma \), it can be shown that the equation for the expected per period profit, assuming that \( Q^* \) units are ordered, is:

\[
EP(Q^*) = (p - s)\mu - (c - s)Q^* - (p + g - s)sL\left(\frac{Q^* - \mu}{\sigma}\right) - K
\] (8.17)
In this expression, \( L(z) \) represents the partial expected value for the standard normal random variable between some value \( z \) and infinity. Values for \( L(z) \) are given in Appendix B.

For Wendell’s Bakery the expected profit per period is:

\[
EP(138) = (.35 - .05)(120) - (.15 - .05)(138) \\
- (.35 + .25 - .05)(20)L\left(\frac{138 - 120}{20}\right) - 15 \\
= .30(120) - .10(138) - .55(20)L(.9) - 15 \\
= 36 - 13.8 - 11(.1004) - 15 = $6.10
\]

**Software Results**

Figure 8.17 gives an Excel spreadsheet that can be used to determine the order quantity for a single-period inventory model in which demand follows a normal distribution.

In this spreadsheet the parameters are entered in column B. The outputs for the problem, including the optimal service level, order quantity, and expected profit, are calculated in cells E5 through E7, respectively.

The worksheet Single-Period (Normal) on the inventory.xls template can be used to determine the optimal inventory policy for single-inventory models in which demand follows a normal distribution. The format for the worksheet is similar to that used in Figure 8.17.

**WENDELL’S BAKERY (CONTINUED)**

As an alternative strategy, suppose the bakery is considering paying the student a commission of $.13 for each donut sold rather than a fixed wage of $15 per evening. Wendell’s wishes to determine how this compensation will affect its inventory decision and whether its daily profits will increase.

**SOLUTION**

In this case, the analysis must be done in a slightly different fashion. In particular, the unit selling price from the bakery’s standpoint is not the full $.35, but the difference between the $.35 selling price and the $.13 commission. Hence,
p = $0.35 - $0.22 = $0.13. For p = $0.22, the optimal inventory quantity, Q*, is chosen so that

\[ F(Q^*) = \frac{0.22 + 0.25 - 0.15}{0.22 + 0.25 - 0.05} = \frac{0.32}{0.42} = 0.7619 \]

This corresponds to a z value of approximately 0.71 and results in an optimal production quantity of

\[ x = \mu + z\sigma = 120 + 0.71 \times 20 = 134 \text{ donuts} \]

To see whether this quantity will increase the bakery’s expected profits, the modified values are substituted into the expected profit equation, giving the following expected per period profit:

\[
\text{EP}(134) = (0.22 - 0.05)(120) - (0.15 - 0.05)(134) \\
\quad - (0.22 + 0.25 - 0.05)(20)L(\frac{134 - 120}{20}) \\
= 0.17(120) - 0.10(134) - 0.42(20)L(0.7) = 20.4 - 13.4 - 8.4(0.1429) \\
= 5.80
\]

Under this plan, the student’s expected compensation can be found by using Equation 8.17, recognizing that from the student’s perspective p = 0.13, c = s = g = K = 0, Q* = 134, \(\mu = 120\), and \(\sigma = 20\). Making the appropriate substitutions gives an expected compensation of $15.23. It is worth noting that the increase in the student’s expected compensation is less than the decrease in the bakery’s expected profit.

This analysis does not account for the possibility that the student’s sales motivation may be greater if compensation is based on commission. If this is the case, the increase in mean donut demand should somehow be estimated and incorporated into the analysis.

**PRACTICAL CONSIDERATIONS OF USING THE SINGLE-PERIOD INVENTORY MODEL**

Whereas the single-period inventory model is easy to solve given the required data inputs, determining the appropriate values for the parameters of a problem is frequently quite difficult. Many businesses do not have any idea what the goodwill cost for an unsatisfied customer might be. Determining the appropriate probability distribution for demand can also be daunting because usually only sales data, which are limited by the amount of inventory available, are observed.

Advanced techniques exist for dealing with these difficulties, but one simplified approach is to solve the problem for a number of different scenarios and determine a “range” of optimal solutions. Once this range has been established, management can select the policy with which it feels most comfortable.

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**8.9 Summary**

In this chapter, we have examined a class of inventory models useful for analyzing goods for which either demand occurs at a known and reasonably constant rate or is random but inventory has a short shelf life. In situations in which a firm purchases goods from a supplier, the economic order quantity (EOQ) model can be used to determine the amount to order. The EOQ solution is quite robust; small errors in estimating parameter values have only minor effects on the EOQ solution.

Firms often offer their customers quantity discounts. Two frequently used discount schedules are the all units schedule and the incremental schedule.
all units discount schedule, simple modifications to the EOQ analysis help determine the optimal order quantity.

In many manufacturing settings, the production lot size model can be used to determine the optimal production quantity for a good. The EOQ model can be considered a special case of the production lot size model, in which goods are supplied at an infinitely rapid rate.

The planned shortage model incorporates a backorder cost into the EOQ model. Items that have a relatively high shortage cost are most likely carried in stock, while items with a relatively high holding cost are most likely to be on backorder.

While some backorders are planned, many are not. In order to determine the necessary safety stock to accommodate a desired service level, a firm can undertake a statistical analysis. The appropriate safety stock is based on whether a cycle service level or a unit service level is desired.

There are several methods firms can use to control their inventories. Review of inventory may be done on a continuous or periodic basis. Order quantities may be fixed or adjusted to bring the inventory up to a specified level.

The single-period inventory (newsboy) model is used in a wide variety of applications in which demand has an exceedingly short shelf life and orders can only be placed once for the goods.

Finally, the control cost associated with an inventory policy should not be overlooked. Although the inventory techniques discussed in this chapter may reduce a firm’s annual inventory expense, there are costs associated with determining inventory policies and controlling the inventory to conform with such policies. Consequently, the methods discussed here generally apply only to A and B type inventory items. For C items, the expense of inventory control generally exceeds the cost savings such control carries.

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**On the CD-ROM**

- Excel spreadsheet for EOQ model
- Excel spreadsheet for determining reorder Point and service level using the cycle Service level approach
- Excel spreadsheet for quantity discount models
- Excel spreadsheet for production lot size model
- Excel spreadsheet for planned shortage model
- Excel spreadsheets for single-period Inventory models
- Excel template for solving inventory problems
- Mathematical Formulas for Inventory Models
- Determining the Reorder Point, R, Corresponding to a Unit Service Level
- Determining the Optimal Order Quantity under an Incremental Discount Schedule
- Derivation of the Planned Shortage Model
- Single-Period Inventory Model
- Production Oriented Inventory Models
- Problem Motivations
- Problems 41–50

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**Additional Resources**

- Allen Appliance.xls
- Allen inventory.xls
- Allen Appliance Cycle Service Level.xls
- Allen inventory.xls
- Farah inventory.xls
- Scanlon inventory.xls
- Sentinel Newspaper.xls
- Wendell’s Bakery.xls
- Inventory.xls
- Appendix 8.2
- Appendix 8.3
- Appendix 8.4
- Appendix 8.5
- Appendix 8.6
- Supplement CD6
- Problem Motivations
- Additional Problems/Cases
APPENDIX 8.1

Using the inventory.xls Template

The template inventory.xls contains nine worksheets. These are:

- EOQ
- All Units Discount
- Incremental Discount
- Production Lot Size
- Planned Shortage
- Single-Period (uniform)
- Single-Period (normal)
- Unit Service Level
- Cycle Service Level

For all worksheets, input data are entered in the yellow colored cells. Outputs are calculated and presented in the light blue cells.

**EOQ WORKSHEET**

Figure A8.1 shows the EOQ worksheet.

![Figure A8.1](image)

The parameters are entered into cells B5 through B10. Inputs include the annual demand in cell B5, the per unit cost in cell B6, the holding cost rate in cell B7, the annual holding cost per unit in cell B8, the ordering cost in cell B9, the
lead time (expressed in years) in cell B10, and the desired safety stock in cell B11.
We note that in cell B8 the holding cost is automatically calculated as $C \times H$.
Should a different value be desired, this value may be entered in the cell.

Outputs for the EOQ model are given in column E. Cell H5 allows one to
calculate a specified value of Q. The resulting outputs are given in cells H6
through H10. This information may be useful, for example, if one wishes to round
off the value of $Q^*$ determined in cell E5 and see the impact of such rounding.

**DISCOUNT MODELS**

The All Units Discount worksheet is shown in Figure 8.9. Figure A8.2 shows the
Incremental Discount worksheet.

![Incremental Discount Worksheet](image)

This worksheet will solve problems with up to eight levels of quantity dis-
counts. Basic data is entered into cells B5 through B12, while the discount data is
entered into cells B17 through C24. Inputs for the Incremental Quantity Discount
worksheet in cells B5 through B11 are similar to those for the EOQ worksheet. In
cell B12, a 1 is entered if the inventory is in discrete units; otherwise a 0 is entered
(or the cell is left empty). For example, if the inventory is cars, we would insert a 1
in cell B12 since one cannot order half a car. Alternatively, if the inventory is flour,
we would put a 0 in cell B12 since it is possible to order half a pound of flour.

Breakpoints are entered in ascending order in cells B17 through B24 for up to
as many levels as there are discounts offered, and the corresponding discounts are
entered in cells C17 through C24. For the 0 discount level (row 16), cell B16 is
preset to a value of 1, while cell C16 is set equal to the original unit cost entered in
cell B6.

Outputs are given in cells E5 through E9. The cost and quantity information
for each level is given in columns D through F beginning in row 16. Note that col-
umn G is hidden as it is used to determine the optimal quantity in cell E5.
Data entry and outputs for the **All Units Discounts** worksheet are similar to those presented for the **Incremental Discount** worksheet. However, for the **All Units Discount** worksheet, since it is not necessary to specify whether the units are discrete, there is no data entry in row 12.

**PRODUCTION LOT SIZE WORKSHEET**

The **Production Lot Size** worksheet is illustrated in Figure 8.12. Inputs and outputs are similar to those for the **EOQ** worksheet, but for this worksheet, the annual production rate must be specified in cell B10.

**PLANNED SHORTAGE WORKSHEET**

The **Planned Shortage** worksheet is illustrated in Figure 8.14. Inputs and outputs are similar to those for the **EOQ** worksheet; however, for this model one must also enter the annual backorder cost in cell B10 and the fixed administrative cost of putting a customer on backorder in cell B11. Note that safety stock is not entered for this model.

The planned shortage model calculates both an optimal order quantity and a backorder level in cells E5 and E6, respectively. The percentage of customers placed on backorder is calculated in cell E12. One can specify values for the order quantity and backorder level in cells H5 and H6, and the resulting inventory information will be determined in the remaining cells in column H.

**SINGLE-PERIOD MODELS**

The formats of the worksheets **Single-Period (uniform)** and **Single-Period (normal)** are similar to those shown in Figures 8.15 and 8.17 respectively. For both worksheets, cost and revenue data are entered in cells B5 through B9. For the **Single-Period (uniform)** worksheet the demand lower bound is entered in cell B10, while the upper bound is entered in cell B11. For the **Single-Period (normal)** worksheet the mean of the demand distribution is entered in cell B10, while the standard deviation is entered in cell B11.

For the **Single-Period (uniform)** worksheet the optimal service level is determined in cell E5, the optimal order quantity is determined in cell E6, and the actual service level (since the order quantity is rounded up) is determined in cell E8. The outputs are the same for the **Single-Period (normal)** worksheet except that cell E8 of this worksheet gives the expected profit.

**SERVICE LEVELS**

The worksheet **Cycle Service Level** is similar to that shown in Figure 8.7. In this worksheet, the inputs are placed in column B. In particular, the mean and standard deviation for lead time demand are inputted in cells B5 and B6, respectively. If one has a desired service level, this information is entered in cell B7 and the corresponding reorder point is calculated in cell E5. If one has a given reorder point, this information is entered in cell B8 and the corresponding cycle service level is calculated in cell E6.

Figure A8.3 shows the worksheet **Unit Service Level**. In this worksheet the mean and standard deviation for lead time demand are inputted in cells B5 and B6, respectively, the order quantity is entered in cell B7, and the desired service level is entered in cell B8. The resulting reorder point and safety stock values are calculated in cells E5 and E6.
1. Culton Hair Salon sells an average of 20 bottles of hair conditioner weekly. There is a $25 cost to place an order with the distributor of the conditioner, and the conditioner costs the salon $2.50 per bottle. The annual holding cost rate is 18%, and the lead-time for delivery is one week. The salon desires a safety stock of five bottles and wishes to determine when an order should be placed and how many bottles it should order.

2. Price-Mart.com stores sells an average of two thousand pairs of the Excite brand of jeans per week. While the jeans normally cost retailers $16.00 per pair, Excite offers customers discounts on all pairs ordered if the order exceeds certain threshold amounts. In particular, the discount pricing schedule is as follows:

<table>
<thead>
<tr>
<th>Order Quantity</th>
<th>Price per Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–1000 pairs</td>
<td>$16.00</td>
</tr>
<tr>
<td>1001–5000 pairs</td>
<td>$15.20</td>
</tr>
<tr>
<td>5,001–10,000 pairs</td>
<td>$14.40</td>
</tr>
<tr>
<td>Over 10,000 pairs</td>
<td>$13.60</td>
</tr>
</tbody>
</table>

Price-Mart.com wishes to determine how many pairs of jeans it should order if the cost of placing an order is $200 and the annual holding cost rate for jeans is 15%.

3. ADM, Inc., a manufacturer of filing cabinets, has an average monthly demand of 500 units for its four-drawer model, ADM-4B. The company’s production facility is capable of producing 2000 filing cabinets per month. Because production of each different model of filing cabinet made by the firm requires different stamping tools, the production setup cost to begin producing the four-drawer models is $2000. The firm estimates that the incremental production cost for the four-drawer model is $40 and that its annual holding cost rate is 20%. ADM wishes to determine how many of the four-drawer models it should manufacture during each production run and the number of production runs for this model that it should schedule over the upcoming year.

4. Playhouse World is distributor for a Victorian-style playhouse manufactured in Thailand. If the firm is out of stock of this playhouse, it offers customers a discount of $50 for each week they must wait for delivery. The administrative cost of processing a backorder is estimated to be $10. The playhouses cost the firm $3500 each and sell for $6000 each. Demand averages two units per month. Due to the fairly high cost of preparing customs documents, the cost of placing an order is estimated to be $1500 and an order takes approximately one month to arrive. Playhouse World estimates that the annual holding cost for the Victorian style playhouse is
4. Appliance Alley is a retailer of brand-name major appliances. One of its best sellers is the Poseidon brand washing machine. Demand for this machine averages 8 units per week. The machines cost Appliance Alley $625 each and sell for $999 each. Appliance Alley estimates that its annual holding cost rate for the washing machines is 18%. The cost of placing an order is $150, and the lead time is three weeks. If Appliance Alley is out of stock of the washing machines, it offers customers a discount of $5 for each day they must wait for their washing machine. The administrative cost of placing a customer on backorder is estimated to be $10.

5. SportWorld.com places orders for Wilson golf balls every other week. Weekly demand for these golf balls averages 50 dozen, and the lead time for delivery of the balls is one week. SportWorld.com wishes to have a safety stock of 10 dozen golf balls to allow for variation in weekly demand. If the inventory at the time an order is placed is 15 dozen golf balls, determine how many dozen golf balls the firm should order.

6. Raul’s Bakery bakes sourdough bread each day. Demand on past days has followed approximately a normal distribution, with an average of 60 loaves and a standard deviation of 12 loaves. The bread sells for $2.50 per loaf and costs $1.00 to produce. Loaves of bread unsold at the end of the day are donated to a food bank and result in the bakery getting an income tax credit of $0.35. If the bakery runs out of sourdough bread, it believes it will suffer a goodwill cost of $5.00 for each loaf that is demanded and is not available.

7. Food Town wishes to determine an optimal order policy for Mariano brand pasta. The store sells an average of 320 one-pound packages per week. The cost of placing an order is $30, and the annual holding cost rate is estimated to be 14%. The pasta costs the store $0.60 per package. Lead time is estimated to be one week, and the store desires a safety stock of 100 packages.

8. Mother Smith’s Pies produces frozen fruit pies for sale to local restaurants. Demand for one of its best sellers, apple, averages 150 pies per day. The company’s production facility is capable of producing 50 pies per hour and operates eight hours a day, seven days a week. The pies cost $2.25 to produce, and Mother Smith’s estimates that its annual holding cost rate is 20%.

9. Appliance Alley is a retailer of brand-name major appliances. One of its best sellers is the Poseidon brand washing machine. Demand for this machine averages 8 units per week. The machines cost Appliance Alley $625 each and sell for $999 each. Appliance Alley estimates that its annual holding cost rate for the washing machines is 18%. The cost of placing an order is $150, and the lead time is three weeks. If Appliance Alley is out of stock of the washing machines, it offers customers a discount of $5 for each day they must wait for their washing machine. The administrative cost of placing a customer on backorder is estimated to be $10.

10. Zeigler’s Lumber Supply sells an average of 15,200 board feet of 2 by 4 lumber weekly. Zeigler’s purchases its 2 by 4 lumber from Western Cascade Wood Products. Western Cascade offers its customers the following all units quantity discount schedule:

<table>
<thead>
<tr>
<th>Order Quantity (in Board Feet)</th>
<th>Cost per Board Foot</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–24,999</td>
<td>$0.26</td>
</tr>
<tr>
<td>25,000–49,999</td>
<td>$0.245</td>
</tr>
<tr>
<td>50,000–99,999</td>
<td>$0.23</td>
</tr>
<tr>
<td>100,000–249,999</td>
<td>$0.22</td>
</tr>
<tr>
<td>250,000–999,999</td>
<td>$0.21</td>
</tr>
<tr>
<td>1,000,000 or more</td>
<td>$0.19</td>
</tr>
</tbody>
</table>

The annual holding cost rate for the lumber is 16%, and the cost of placing an order for the lumber is estimated to be $250. Lead time is three weeks, and Zeigler’s desires a safety stock of 8000 board feet. Determine the optimal inventory policy of 2 by 4’s for Zeigler’s Lumber Supply.

11. Bank Drugs wishes to determine how many tablets of a new arthritis medication it should order. Its policy is to order from the manufacturer every other week and maintain a safety stock of 120 tablets. Based on past weeks’ sales, Bank estimates that it sells 800 tablets of the medication each week. The lead time for delivery of the medication is three days. If its current inventory is 350 tablets, determine how many tablets of the medication it should order. Assume Bank is open seven days a week.

12. It is October 15, and Furr’s Stationary must decide how many calendars it should order from the World Wildlife Federation. The calendars cost the company $4.25, and Furr’s sells them for $9.50. The calendars will arrive on November 1, and demand during the period between November 1 and Christmas is estimated to follow a normal distribution with a mean of 250 units and a standard deviation of 40 units. Any calendars that remain after Christmas will be marked down to $2.00 and sold at Furr’s annual after Christmas sale. If Furr’s runs out of calendars before Christmas, it estimates it suffers a goodwill cost of $1.50 for each calendar demanded when it is out of stock. If Furr’s can only place one order for the calendars and there is a $20 cost of placing an order, determine:

a. How many calendars it should order.

b. The expected profit it will earn on the calendars.

13. Demand for Stick disposable razors at Buyright Drugs averages seven packages per day. The razors cost Buyright $0.80 per package and sell for $1.49. Buyright uses a 20% annual holding cost rate and estimates the cost to place an order for additional razors at $25.

Problems
stock of 15 packages. The lead time for delivery is five days. Determine the following:

a. The optimal inventory policy (order quantity and reorder point) for Stick razors.
b. The number of days between orders (cycle time).
c. The total annual inventory cost (holding, ordering, and procurement) and the projected annual net profit of this policy.

What assumptions did you make regarding demand in solving this problem?

14. How would your answers to problem 13 change if Stick requires its customers to purchase razors in gross units (multiples of 144) and Buyright desires a safety stock of 20 razors?

15. OfficeHQ is a discount retailer of office goods. One of its most popular products is the Stick brand pen, packaged six to a box and retailing for $0.95. OfficeHQ is open five days a week, 52 weeks a year.

Daily demand for Stick pens is reasonably constant, averaging 65 boxes. The cost for OfficeHQ to place an order is $30, and the firm uses an annual inventory holding cost rate of 22%. Lead time for delivery is one week, and OfficeHQ desires a safety stock of 100 boxes of pens. If OfficeHQ must order in increments of 100 boxes, determine the following:

a. The optimal inventory policy (order quantity and reorder point) for Stick pens.
b. The number of calendar days between orders (cycle time).
c. The total annual inventory cost (holding, ordering, safety stock, and procurement) and the projected annual net profit of this policy.

16. Suppose that in problem 15 the daily demand for Stick pens averages 75 boxes (instead of 65).

a. What is the optimal order quantity for Stick pens?
b. If OfficeHQ uses the order policy determined in problem 15, determine the difference in total annual variable inventory costs between this policy and the optimal policy found in part (a).

17. OfficeHQ carries boxes of Disco floppy diskettes. Because the diskettes come in different formats, OfficeHQ has decided to use a periodic review policy, in which it places an order with Disco once every three weeks.

Weekly demand for Disco 3 1/2" diskettes at OfficeHQ averages 45 boxes. The lead time for delivery is approximately one week, and OfficeHQ desires a safety stock of 30 boxes. OfficeHQ uses an annual holding cost rate of 20% for the diskettes. If the inventory level at the time OfficeHQ places its next order with Disco is 55 boxes, determine its optimal order quantity.

18. Scanlon Plumbing Corporation distributes American Consolidated lavatories. Demand for the basic China White Oval Model 2634 averages 19 units a week. The lavatories cost Scanlon $22.50 each and sell for $35.75. Unfortunately, approximately 5% of the lavatories ordered by Scanlon are either defective or damaged during shipment. Therefore, Scanlon needs 20 (= 19/0.95) units a week to meet its demand.

The company uses a periodic review policy with American Consolidated and orders once every four weeks. Lead time for delivery is two weeks, and Scanlon desires a safety stock of 50 units. If the inventory level of the lavatories at the time of the next order is 35 units, determine the optimal inventory policy.

19. GROW Garden Center sells Raincloud automatic sprinkler valves. The valves cost GROW $8.75 each, and GROW uses an annual holding cost rate of 24%. The cost to place an order with Raincloud is approximately $30.

Demand over the past eight weeks has been as follows:

<table>
<thead>
<tr>
<th>Week</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>22</td>
</tr>
<tr>
<td>5</td>
<td>18</td>
</tr>
<tr>
<td>6</td>
<td>22</td>
</tr>
<tr>
<td>7</td>
<td>18</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
</tr>
</tbody>
</table>

GROW uses a simple eight-week moving average to forecast average annual demand and average lead time demand. Lead time for delivery is two weeks, and GROW desires a cycle service level of 96%. GROW estimates that lead time demand follows a normal distribution with a standard deviation of 5.45 units. On the basis of this information, determine:

a. The optimal inventory policy (order quantity and reorder point) for the sprinkler valves.
b. The number of calendar days between orders (cycle time).
c. The total annual inventory cost (holding, ordering, procurement) for this policy.

20. Bryan’s Office Supply sells the Harrington 2000 automatic stapler. Demand for the staplers averages 24 units per week. The staplers cost Bryan’s $17.25 each, and Bryan’s uses a 22% annual holding cost rate. The cost to place an order with Harrington is $45, and the lead time for delivery is two weeks. Bryan’s desires a safety stock of 15 staplers.

Harrington offers its customers the following all unit quantity discount pricing schedule:

<table>
<thead>
<tr>
<th>Number Ordered</th>
<th>Discount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–199</td>
<td>none</td>
</tr>
<tr>
<td>200–399</td>
<td>4%</td>
</tr>
<tr>
<td>400–699</td>
<td>6%</td>
</tr>
<tr>
<td>700–999</td>
<td>8%</td>
</tr>
<tr>
<td>1000–4999</td>
<td>11%</td>
</tr>
<tr>
<td>5000+</td>
<td>15%</td>
</tr>
</tbody>
</table>

Bryan’s never orders an amount greater than a 35-week supply for any product. Assuming that holding costs are discounted, determine the following:

a. The optimal inventory policy (order quantity and reorder point) for Harrington staplers.
b. The number of calendar days between orders (cycle time).
c. The total annual inventory cost (holding, ordering, procurement, and safety stock) for this policy.

What assumptions did you make to solve this problem?

21. Archer Pharmaceuticals manufactures Tranquility brand sleeping pills, which have an average weekly demand of 25,000 bottles. The Archer factory operates 10 hours a
day, six days a week. The production line at Archer Pharmaceuticals can produce 10,000 bottles of Tranquility daily. Archer’s policy is to make a batch of Tranquility and, after building up sufficient inventory, use the production line to produce other products.

The company estimates that setup for producing Tranquility pills takes about two hours and costs $325. It also estimates that the annual holding cost for a bottle of Tranquility is $0.55.

a. What is the optimal production batch size and the resulting total annual inventory holding and production setup cost of this policy?
b. What is the length of a production run in hours (including setup time)?
c. What is the number of calendar days between the start times of successive production runs?
What assumptions did you make in solving this problem?

22. Bee’s Candy manufactures a variety of candy bars in a number of different sizes. One of its more popular products is the three-ounce Smirk bar. Bee’s forecasts that demand for this product should be fairly constant over the next year, totaling 21 million bars.

The candy production line, which operates 24 hours a day, 365 days a year, is capable of producing two candy bars per second. A production setup for the three-ounce Smirk bar takes 50 minutes and costs approximately $450. The annual holding cost of a three-ounce Smirk bar is estimated as $0.12.

a. What is the optimal production batch size and the length of a production run in hours (including setup time)?
b. What is the total annual inventory holding and production setup cost for this policy?
c. What is the number of days between the start times of successive production runs?
d. How would your answers to part (a) change if the annual holding cost decreased to $0.10 per bar?

23. Click Pix, a large discount camera shop in New York City is open six days a week, 52 weeks a year. The store recently began carrying Sonic model PS58 camcorders which cost $520.00 each and retail for $649.99. Sales average 60 units per week.

The cost of placing an order with Sonic is $90, and the lead time is seven working days. The store estimates that the lead time demand follows a normal distribution with a mean of 70 units and a standard deviation of 250. Click Pix uses an annual holding cost rate of 18% for the camcorders. Ideally, it would like to run out of the camcorders during at most one inventory cycle per year. Given this goal, determine the following:

a. The optimal inventory policy (order quantity and reorder point) for Sonic camcorders.
b. The number of working days between orders (cycle time).
c. The total annual inventory cost (holding, ordering, procurement, and safety stock) for this policy and the projected annual profit for this policy.

24. How would your answers to problem 23 change if Sonic offered Click the following all units discount schedule?

<table>
<thead>
<tr>
<th>Order Quantity</th>
<th>Discount</th>
</tr>
</thead>
<tbody>
<tr>
<td>200–599</td>
<td>5.0%</td>
</tr>
<tr>
<td>600–999</td>
<td>7.5%</td>
</tr>
<tr>
<td>1000 or more</td>
<td>9.0%</td>
</tr>
</tbody>
</table>

25. Masks-R-Us sells Halloween masks at a kiosk in the local mall. The store is open only during the month of October. Masks are imported from Asia and cost the store $3.45 each; they retail for $9.95 each. Any masks left in inventory after Halloween are sold to a merchandise closeout specialist at a price of $1.80 each.

Masks-R-Us estimates that if it runs out of Halloween masks, it will suffer a customer goodwill loss of $15. The company further estimates the cost of rent, utilities, labor, insurance, and so on, to operate the kiosk is approximately $4000 for the month.

Based on past sales, Masks-R-Us estimates that demand for its masks during the month of October will be approximately normally distributed with a mean of 900 masks and a standard deviation of 100 masks. Because the masks are imported from Asia, Masks-R-Us must place its order with the mask manufacturer in May.

Determine the number of masks Masks-R-Us should order and the expected profit or loss it can expect to earn if it follows this policy.

26. Jackson Mint produces collectible plates. It has recently contracted with the estate of well-known artist, Skip Gunther, to produce a series of five commemorative plates bearing copies of the artist’s most famous pictures. The plates will be sold by subscription at a cost of $275 plus shipping and handling for the entire five-plate series. Jackson estimates the cost of producing each plate in the series at $12.50 plus a $9.75 royalty that Jackson has agreed to pay the Gunther estate.

Jackson intends to mount a $175,000 nationwide advertising campaign promoting these plates as a “limited edition”; that is, Jackson will specify the number of plates it will produce and limit production to that amount. Any unsold plates will be destroyed. Based on its previous experience with such products, Jackson estimates that customer demand for the series will be approximately normally distributed, with a mean of 1800 and a standard deviation of 250.

If Jackson has more subscribers than plate series, it will refund subscribers’ money with a note explaining that the series has been oversubscribed. Reasoning that the unsatisfied customers will be more willing to subscribe to Jackson’s future offerings, it estimates that each unsatisfied customer will earn the company an average of $40 in discounted future profits.

a. What is the optimal number of Skip Gunther plate series Jackson should produce?
b. What expected profit or loss will Jackson earn if it follows the production policy found in part (a)?
c. Comment on any assumptions you made to solve this problem.
27. Scott Stereo sells personal tape players. One of its more popular sellers is the Sonic Walkperson. These units cost Scott $18.55 each and retail for $32.95. Weekly demand averages 12 units. The cost of placing an order with Sonic is $25, and lead time is two weeks. Scott uses a 20% annual inventory holding cost rate.

If Scott is out of stock of the Walkperson, it estimates that it will suffer a customer goodwill cost of $2 for each week a customer must wait for the Walkperson to arrive. A fixed administrative cost of $0.50 is associated with each backordered customer. Determine the following:

a. The optimal inventory policy (order quantity and reorder point) for the Walkpersons.

b. The number of calendar days between orders (cycle time).

c. The percentage of customers who will be placed on backorder.

d. The total annual inventory cost (holding, ordering, shortage, and procurement) for this policy and the projected total annual profit for this policy.

28. Scott Stereo orders Dutch brand cassette tapes once every three weeks. Weekly demand for the T-60 format is approximately normal with a mean of 200 units and a standard deviation of 30 units. The tapes cost Scott $0.65 each and sell for $1.25. Lead time for delivery is one week. Scott desires a safety stock that will give the store a cycle service level of 98%. If Scott’s inventory level for the Dutch T-60 tapes is at 230 units when it places an order, determine:

a. The order quantity for the tapes.

b. The desired safety stock.

c. The optimal order quantity for the tapes.

d. The total annual inventory cost (holding, ordering, shortage, and procurement) for this policy and the projected total annual profit for this policy.

29. Pete’s Coffee is a local shop that roasts and packages its own coffee. The store purchases its Colombian coffee beans from Valdez Importing Company (VIC). VIC offers its customers the following discount pricing schedule for Colombian coffee:

<table>
<thead>
<tr>
<th>Order Quantity</th>
<th>Price per Pound</th>
</tr>
</thead>
<tbody>
<tr>
<td>under 1000 lbs.</td>
<td>$1.25</td>
</tr>
<tr>
<td>1000–3000 lbs.</td>
<td>$1.12</td>
</tr>
<tr>
<td>3000–6000 lbs.</td>
<td>$1.055</td>
</tr>
<tr>
<td>6000–9000 lbs.</td>
<td>$2.99</td>
</tr>
<tr>
<td>over 9000 lbs.</td>
<td>$2.925</td>
</tr>
</tbody>
</table>

Pete’s estimates its annual demand for Colombian coffee beans at 62,000 pounds. The annual holding cost rate for a pound of unroasted coffee beans is estimated at 30%. The cost to place an order with VIC is $125, and the lead time is four weeks. Pete’s desires a safety stock of 1000 pounds.

a. What is the optimal order quantity if the discount pricing schedule is an all units schedule?

b. What is the optimal order quantity if the discount pricing schedule is an incremental schedule?

c. What is the reorder point for Colombian coffee?

What assumptions did you make in solving this problem?

30. Pete’s Coffee is planning to stock the Melvitta brand espresso maker. Pete’s estimates the annual demand for this coffee maker to be 180 units. The machines, which retail for $189 each, are imported from Italy and cost Pete’s $95 each. The cost to place an order with Melvitta is $150, and the lead time is an estimated five weeks. Pete’s uses a 30% annual holding cost rate for the coffee makers.

Under the second plan, Pete’s will offer backordered customers three free pounds of coffee. This will cost $4.25, and Pete’s estimates an additional customer goodwill loss of $15 for each week a customer must wait for the espresso maker.

Under the second plan, Pete’s will offer backordered customers one free pound of coffee. This will cost $12.75 but will reduce the additional goodwill loss to $3 for each week a customer must wait for the espresso maker.

What assumptions did you make in solving this problem?

31. Pete’s Coffee is considering using one of two customer satisfaction plans to deal with the possibility of stockouts of the espresso maker. Under the first plan, Pete’s will offer backordered customers one free pound of coffee. Under the first plan, Pete’s will offer backordered customers one free pound of coffee. Under the first plan, Pete’s will offer backordered customers one free pound of coffee. Under the first plan, Pete’s will offer backordered customers one free pound of coffee.

Pete’s is considering using one of the following customer satisfaction plans to deal with the possibility of stockouts of the espresso maker:

a. A 26% annual inventory holding cost rate.

b. A 20% annual inventory holding cost rate.

c. Using this policy, what is Pete’s expected annual profit on the espresso machines?

32. Business Supply Company, Inc. (BSC) is a local distributor of the NCQ electronic cash register. The cash registers cost BSC $320 each, and (neglecting inventory costs) BSC estimates it earns a profit of $80 on each cash register sold. The cost of ordering the cash registers from NCQ is $100, and BSC uses an annual inventory holding cost rate of 20%.

If BSC runs out of the cash registers, it estimates it will suffer a customer goodwill cost of $50 for each week a customer must wait. There is also a fixed administrative cost of $1.50 to process a backorder. Weekly demand averages 75 units, and the delivery lead time is two weeks. Determine the following:

a. The optimal inventory policy (order quantity and reorder point) for NCQ cash registers.

b. The number of weeks between cash register orders.

c. The percentage of customers who will be placed on backorder.

d. BSC’s annual profit on the cash registers.

33. Clothesline, Inc. is a retailer of moderately priced women’s clothing. Clothesline’s policy is to give customers a $5 gift certificate if they request an out-of-stock advertised item. This certificate is estimated to cost the company only $3, but it avoids a loss in customer goodwill.
Clothesline is considering selling a line of scarves produced by a noted fashion designer. Because of long lead times, Clothesline can only place one order for the scarves. The scarves cost Clothesline $6.25, and it plans to retail them for $10.95. Clothesline estimates that total demand at its 55 stores during the selling season will follow approximately a normal distribution, with a mean of 8000 units and a standard deviation of 1500 units. Any scarves left in inventory at the end of the selling season will be marked down to a clearance price of $4.95 and sold quickly.

On the basis of this information, determine how many scarves Clothesline should order. Comment on any assumptions you made to solve this problem.

34. Stefani Foods produces fresh pasta products for sale through local supermarkets. One product that is quite popular is fresh linguini and clam sauce. This item has a shelf life of two weeks and costs Stefani $1.19 per unit to produce. The product retails at a suggested price of $1.99 and is sold to supermarkets at a wholesale price of $1.46.

Many supermarkets receive only one delivery per week of Stefani products. In these instances, the delivery agent is instructed to pick up any unsold linguini to return to the company. The supermarkets receive full credit on these returns, and the returned merchandise is sold in the Stefani owned Thrift Store at a retail price of $0.83 per unit.

Van’s Supermarket estimates that weekly demand for the linguini follows a uniform distribution between 160 and 239 units. Stefani delivers to Van’s once a week.

Determine the optimal stocking quantity for the linguini and clam sauce. This item has a shelf life of two weeks and costs Stefani $1.19 per unit to produce. The product retails at a suggested price of $1.99 and is sold to supermarkets at a wholesale price of $1.46.

Many supermarkets receive only one delivery per week of Stefani products. In these instances, the delivery agent is instructed to pick up any unsold linguini to return to the company. The supermarkets receive full credit on these returns, and the returned merchandise is sold in the Stefani owned Thrift Store at a retail price of $0.83 per unit.

Van’s Supermarket estimates that weekly demand for the linguini follows a uniform distribution between 160 and 239 units. Stefani delivers to Van’s once a week.

Determine the optimal stocking quantity for the linguini and clam sauce. This item has a shelf life of two weeks and costs Stefani $1.19 per unit to produce. The product retails at a suggested price of $1.99 and is sold to supermarkets at a wholesale price of $1.46.

35. The Circle 7 convenience store has been receiving deliveries of Royal Cola once a week. Weekly demand for the cola averages 60 cases and follows a normal distribution with a standard deviation of 12 cases. When the delivery truck arrives, the store orders enough cola so that the cycle service level is 99%. Because the orders arrive on a regular basis, there is no ordering cost to the store.

The annual holding cost rate for a case of cola is estimated at 25%.

Royal Cola has just instituted a quantity discount schedule. Instead of charging stores like Circle 7 the normal price of $4.25 per case, Royal is offering the following all units discount schedule:

<table>
<thead>
<tr>
<th>Number of Cases Ordered</th>
<th>Price per Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>under 100</td>
<td>$4.25</td>
</tr>
<tr>
<td>100–199</td>
<td>$4.00</td>
</tr>
<tr>
<td>200–499</td>
<td>$3.80</td>
</tr>
<tr>
<td>500–899</td>
<td>$3.70</td>
</tr>
<tr>
<td>900 or more</td>
<td>$3.60</td>
</tr>
</tbody>
</table>

If Circle 7 wants to take advantage of the quantity discounts being offered by Royal Cola, it will have to cancel its regular delivery schedule. In this case, it estimates that the cost to place an order with Royal Cola will be $35 and lead time will be one week. Circle 7 will continue to desire a cycle service level of 99%.

a. If Circle 7 continues with the regular delivery schedule and has 22 cases in inventory when the next delivery arrives, how many cases will be delivered to the store?

b. What is the annual inventory cost (holding, safety stock, and procurement) of the regular delivery schedule?

c. If Circle 7 decides to take advantage of the quantity discount schedule, how many cases should it order?

d. What is the annual inventory cost (holding, ordering, safety stock, and procurement) associated with the firm using the quantity discount schedule?

e. What is the reorder point for the answer in part (c)?

f. Do you recommend that Circle 7 take advantage of the quantity discount schedule? Why or why not?

g. How would your answers to parts (c), (d), (e), and (f) change if Circle 7 had enough room to store only 250 cases of Royal Cola?

36. Clark Equipment distributes the Clanton Model 406 bread slicer used in bakeries. The slicers cost $250 each, and Clark sells them to net $306 after marketing and related expenses. The company estimates that the annual demand for this product is 450 units.

The policy at Clark has been never to allow stockouts intentionally, and the company has carried a safety stock of 30 units in order to protect against such occurrences. Because of mounting fiscal pressure, however, Clark is considering eliminating the safety stock for the bread slicers and adopting a policy that allows for stockouts.

Clark estimates that it will suffer a customer goodwill loss of $25 per week for each week a customer must wait for a backordered bread slicer. Clark also believes that there is an administrative cost of $30 in handling a backorder and that adopting such a policy would result in a 4% decrease in annual sales of the bread slicer.

Clark uses a 15% annual inventory holding cost rate, and the cost of ordering slicers from Clanton is $90. Delivery lead time is 10 working days, and Clark is open 250 days a year.

a. Determine the optimal order quantity, reorder point, and annual profit under the current policy of not intentionally allowing backorders.

b. Determine the optimal order quantity, reorder point, and annual profit if Clark intentionally allows for backorders.

c. What is your recommendation to management as to whether Clark should intentionally allow for backorders? Justify this recommendation.

37. Microvision currently purchases a particular computer chip from IMTEL for $12.42 each, for use in its PC computers. Microvision’s annual demand for the chip is
estimated at 140,000 units, and it has a safety stock requirement of 600 chips. Because of high security expense, the cost to place an order with IMTEL is estimated at $1300 and lead time is 20 working days. The company operates 310 days per year.

Instead of purchasing from IMTEL, Microvision can sign a licensing agreement to manufacture the chips itself. The licensing agreement will cost Microvision $5000 per year. If Microvision signs the agreement, it estimates it can produce the chips on its own assembly line at a cost of $11.60 per chip (not including the licensing or inventory costs). Setup time will take two days.

The production setup cost for making these chips is $15,000, and the production line is capable of manufacturing 2000 chips a day, 310 days a year. If the company produces the chips in-house, it will no longer require any safety stock. Microvision uses a 24% annual holding cost rate.

a. What are the optimal order quantity, reorder point, number of days between orders (cycle time), and total annual inventory cost (holding, ordering, safety stock, and procurement) if Microvision purchases the chips from IMTEL?
b. What are the optimal batch size, length of a production run in days (including production setup time), number of days between the start of successive production runs, and total annual inventory cost (holding, setup, licensing, and production) if Microvision begins producing the chips in-house?
c. What is your recommendation to management as to whether Microvision should begin in-house production of the chips? Justify this recommendation.

38. Consider the data from problem 37. Suppose IMTEL has decided to offer incremental price discounts on chips sold to Microvision. In particular, the new pricing schedule is as follows:

<table>
<thead>
<tr>
<th>Order Quantity</th>
<th>Discount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–4,999</td>
<td>none</td>
</tr>
<tr>
<td>5,000–24,999</td>
<td>5%</td>
</tr>
<tr>
<td>25,000–49,999</td>
<td>8%</td>
</tr>
<tr>
<td>50,000–99,999</td>
<td>10%</td>
</tr>
<tr>
<td>100,000 or more</td>
<td>13%</td>
</tr>
</tbody>
</table>

a. What is Microvision’s optimal order quantity for chips under this pricing policy if it continues to purchase chips from IMTEL?
b. Determine the total annual cost of this policy to Microvision.

39. Johansen’s Ice Cream Shoppe purchases fresh-baked waffle cones from the Myra Cone Company. The cones cost Johansen $0.28 each and are delivered once each day. Johansen’s charges customers who want their ice cream in a waffle cone an extra $0.40. If Johansen’s runs out of waffle cones, it estimates that it suffers a customer goodwill loss of $0.75 for each additional customer request for a waffle cone. Unsold waffle cones are ground up and used as toppings on the frozen yogurt sold by the company. Johansen’s therefore estimates the salvage value of unsold cones at $0.06. Johansen’s also estimates that daily demand for the waffle cones follows approximately a normal distribution with a mean of 70 units and a standard deviation of 16 units. Determine how many waffle cones Johansen’s should purchase each day from Myra Cone.

40. Frank’s Garden Center sells McMurray riding mowers. McMurray has found itself with an excess inventory of Model 412 mowers and is temporarily offering dealers a discount of $75 off its normal wholesale price of $725. Frank’s sells the Model 412 at a retail price of $895 and estimates that its demand during the upcoming selling season will be between one and five units, with the following probabilities:

<table>
<thead>
<tr>
<th>Demand</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 unit</td>
<td>.25</td>
</tr>
<tr>
<td>2 units</td>
<td>.20</td>
</tr>
<tr>
<td>3 units</td>
<td>.10</td>
</tr>
<tr>
<td>4 units</td>
<td>.30</td>
</tr>
<tr>
<td>5 units</td>
<td>.15</td>
</tr>
</tbody>
</table>

Unfortunately, Frank’s must decide how many mowers to purchase at the temporary price discount before the selling season begins. Once the selling season starts, if demand for the McMurray mower at Frank’s exceeds the number ordered at the discounted price, Frank’s must pay the normal wholesale price of $725 per unit. Any purchased mowers that Frank’s is not able to sell during the normal selling season can be sold for $595 at the annual clearance sale. How many mowers should Frank’s purchase from McMurray at the temporary price discount?

PROBLEMS 41–50 ARE ON THE CD

CASE STUDIES

**Case I: TexMex Foods**

TexMex Foods operates a plant in Irving, Texas, for manufacturing taco sauce used in fast-food restaurants. The sauce, which is packaged in plastic containers, is made from a special recipe that includes tomato concentrate, onions, and chile peppers that TexMex purchases from various suppliers. The plant operates 365 days a year, and TexMex uses an annual holding cost rate of 18%.
**Tomato Concentrate**

TexMex Foods purchases its tomato concentrate from Hunt Farms. The company requires 2500 gallons of concentrate per day to manufacture this sauce. Hunt Farms offers customers the following all units price discount schedule:

<table>
<thead>
<tr>
<th>Number of Gallons Ordered</th>
<th>Price per Gallon</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–9999</td>
<td>$3.12</td>
</tr>
<tr>
<td>10,000–49,999</td>
<td>$3.08</td>
</tr>
<tr>
<td>50,000–124,999</td>
<td>$3.02</td>
</tr>
<tr>
<td>125,000–249,999</td>
<td>$3.01</td>
</tr>
<tr>
<td>250,000 or more</td>
<td>$2.96</td>
</tr>
</tbody>
</table>

The shelf life of the concentrate is 80 days, and the ordering cost is $750. Orders must be placed in 1000-gallon increments. The company desires a safety stock of 10,000 gallons, and the lead time for delivery is 10 days. Management wishes to determine the optimal order quantity for the concentrate as well as the reorder point.

**Onions**

In the cooking process, TexMex requires 6000 pounds of onions daily. Onions cost the company $0.15 per pound, and the ordering cost is $180. Lead time for delivery is three days, and the company desires a safety stock equal to one day’s usage. Management wants to determine the optimal order quantity for onions as well as the reorder point.

**Chile Peppers**

TexMex also needs an estimated 2000 pounds of chile peppers daily. The peppers cost TexMex $0.37 per pound. Order cost, including transportation, is $1500. Lead time is normally two weeks but may vary somewhat. Because of this variability, the company estimates that the lead time demand for chile peppers follows approximately a normal distribution, with a mean of 28,000 pounds and a standard deviation of 4000 pounds. Management wants to determine the optimal order quantity, reorder point, and safety stock for chile peppers to meet a desired cycle service level of 99.5%.

**Plastic Containers**

TexMex packages the sauce in one-ounce plastic containers it buys from Union Chemical at $0.003 per unit. The ordering cost is $120. TexMex is contemplating leasing a machine to make the containers. The yearly lease cost of the machine is $45,000, and the production setup cost is $260. The machine can produce 1 million containers per day at a per unit cost of $0.0027 (excluding leasing, inventory holding, and production setup costs). The company estimates that it requires 450,000 containers per day. Management wants to determine whether it should continue purchasing containers from Union Chemical or begin in-house production and what the optimal order quantity or production lot size should be.

Prepare a detailed management report addressing each of the concerns facing TexMex Foods. Include in your report supporting graphs and charts as well as appropriate “what-if” analyses.

**Case 2: Rodman Industries**

Rodman Industries of Barstow, California, sells among other items, specialized tires for off-road vehicles (ORV). A new ORV model requiring tires slightly larger than normal is being developed by Pacific Star Enterprises in nearby Apple Valley. Pacific Star and Rodman have done business together for years, and Rodman has contracted with Pacific Star to supply tires for the new vehicle.

Rodman’s staff analyst has estimated that Pacific Star’s weekly demand will follow approximately a normal distribution, with a mean of 2000 tires and a standard deviation of 100 tires. Rodman charges Pacific Star $25 per tire, and Rodman’s holding costs are figured at 20% per year.

**Manufacturing/Purchasing Options**

Rodman has three choices available to it for supplying tires to Pacific Star:

1. Rodman can convert production line 3 to manufacture the tires. The equipment on this line can be converted at a cost of $150,000 to produce the new tire. This production line will have a maximum production rate of 4000 tires per week. It takes roughly one week to set up between production runs, and each setup costs approximately $4000. Unit production costs (raw materials and labor) are $12 per tire.

2. Rodman can convert production line 5 to manufacture the tires. This line is capable of producing only 1800 tires per week. It is very reliable, however, and, after a conversion cost of $75,000, the line is expected to run without failure. Although this option means that Rodman could supply Pacific Star with only 1800 tires per week, Pacific Star has indicated that it would accept this quantity, if necessary. Again, unit production costs (raw materials and labor) are $12 per tire.

3. Rodman can purchase tires from Hiro Inc., a Japanese firm, and import them to its San Pedro warehouse for distribution directly to Pacific Star. Reorder costs, which include some substantial shipping fees, are estimated at $10,000 per order, and shipping time is consistently two weeks from the time an order is
placed. Hiro charges $14 per tire but offers the following all units discount pricing schedule:

<table>
<thead>
<tr>
<th>Order Quantity</th>
<th>Discount</th>
</tr>
</thead>
<tbody>
<tr>
<td>under 5000</td>
<td>none</td>
</tr>
<tr>
<td>5000–9999</td>
<td>7.5%</td>
</tr>
<tr>
<td>10,000 or more</td>
<td>15.0%</td>
</tr>
</tbody>
</table>

Safety Stock
While selecting option 2 allows Rodman to maintain no safety stock, for options 1 and 3 Rodman should have enough safety stock to maintain at least a 90% cycle service level.

Case 3: Mr. Pretzel

Mr. Pretzel sells soft pretzels to movie theaters, skating rinks, and snack bars. The company bakes the pretzels in its factory and delivers them fresh daily to its various accounts. Mr. Pretzel has recently signed a contract to purchase 50 pretzel vending machines. Management feels that these machines will enable the company to sell its pretzels in new locations, thus expanding company revenue.

Mr. Pretzel’s marketing department is busy trying to determine suitable locations for the vending machines. It has decided that, at a minimum, pretzel demand at a vending machine location must average at least 25 units per day. Each vending machine is capable of holding 140 pretzels. The marketing department is interested in determining what compensation it should offer the owners of the sites at which the vending machines will be placed.

According to the company’s accounting department, the cost of producing a soft pretzel (fixed and variable) is approximately $.32. The pretzels to be sold in the vending machines are sealed in plastic, adding $.005 to their cost. They are delivered daily and sell for $.75. Any pretzels left in the vending machine from the previous day are removed and returned to the factory. There they are ground up and sold for use in cattle feed, for which the company receives approximately $.02 in net revenue from each returned pretzel. The cost of delivering new pretzels and removing old ones is estimated to be $4.20 per vending machine per day. An additional capital cost of $2.50 per day is associated with each vending machine.

The Report
On the basis of the given information, prepare a business report for Rodman suggesting a policy that will optimize total weekly profit for the company. Assume that this is a three-year project (156 weeks) and that conversion costs can be amortized at a constant rate over this time period. Include in your report any assumptions you made (or model assumptions you violated) in doing your analysis. The report should contain a table giving, for each option, the optimal order or production quantity, the reorder or setup point, and the total weekly revenue, costs, and profit.

If the vending machine runs out of pretzels, there is some likelihood that a dissatisfied customer may kick the machine, causing some damage. Since the company has not had any operating history regarding these machines, it is uncertain of the goodwill and potential damage cost of not having enough pretzels in the machine to satisfy all customers.

The company has three different compensation schemes for site owners:

1. Pay the site owner a fixed revenue of $3 per day.
2. Pay the site owner a commission of $.08 per pretzel sold.
3. Lease the vending machine to the site owner with the stipulation that Mr. Pretzel will service the machine. Mr. Pretzel will charge the site owner $3.25 per day for leasing the machine and $.45 apiece for the pretzels. Mr. Pretzel will give no credit for unsold pretzels, and the pretzels have no salvage value to the site owner.

Prepare a management report indicating the expected daily profit to Mr. Pretzel for each compensation plan, assuming daily demand follows a Poisson distribution with means, $\lambda$, of either 25, 50, 75, or 100 units. Use the normal distribution approximation to the Poisson distribution ($\mu = \lambda$ and $\sigma = \sqrt{\lambda}$). Assume that if Mr. Pretzel leases the vending machine to the site owner, the site owner will order the optimal number of pretzels from the company. Do the analysis for at least three possible goodwill costs, including $.05, $.25, and $.75.